



**ADULT
NUMERACY**
INSTRUCTION PROJECT

Building on *Foundations for Success*

Guidelines for Improving
Adult Mathematics Instruction

Building on *Foundations for Success:* Guidelines for Improving Adult Mathematics Instruction

Prepared under contract to
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Executive Summary

The National Mathematics Advisory Panel (NMAP) was established in 2006, with the charge of determining how to “foster greater knowledge of and improved performance in mathematics among American students” based on “the best available scientific evidence” (Executive Order No. 13,398, 2006). The report of the Panel’s work, *Foundations for Success: The National Mathematics Advisory Panel Final Report* (2008), offered a series of recommendations for improving mathematics education and learning among K–12 students. The recommendations focus on seven broad areas: curricular content; learning processes; teachers and teacher education; instructional practices; instructional materials; assessment; and research policies and mechanisms.

Recognizing the paucity of research available on adult numeracy instruction, the U.S. Department of Education, Office of Vocational and Adult Education (OVAE) initiated the *Strengthening America’s Competitiveness Through Adult Math Instruction* project. This project seeks to determine (1) *what to teach in adult numeracy instruction*, (2) *how to teach it*, and (3) *how to teach teachers to teach it*. These questions form the basis for this report. To begin this work, OVAE proposed to examine the *Foundations for Success* report (herein called “the NMAP report”) to determine if any of its findings or recommendations could apply to mathematics instruction for adults.

OVAE contracted with MPR Associates, Inc. and its partners, the Center for Literacy Studies at the University of Tennessee, Rutgers University, and TERC, to analyze the NMAP report, guided by subject matter experts in the fields of mathematics education and mathematical cognition and learning, to determine its applicability to adult education. This analysis, along with a review of research on adult education, adult mathematics instruction, and numeracy education, among other relevant materials, is the foundation for the guidelines for adult mathematics instruction presented here.¹ These guidelines reflect adults’ goals in seeking basic skills instruction, including: managing daily life, preparing for the General Educational Development (GED) test, seeking or advancing in employment, and pursuing advanced education and training (Comings, 2007; Ginsburg, Manly, & Schmitt, 2006; Tamassia, Lennon, Yamamoto, & Kirsch, 2007).

¹ This analysis and results of the literature review, with appropriate citations, are presented in the body of this report.

EXECUTIVE SUMMARY

The guidelines for adult mathematics instruction, drawn from relevant NMAP recommendations and research, are discussed in the following sections. In essence, these guidelines sketch a vision of improved mathematics instruction for adults that:

- Incorporates important mathematics topics relevant to the lives and goals of adults.
- Proceeds in an integrated, coherent progression that may vary according to adult students' goals and skill levels.
- Promotes a multifaceted definition of mathematical proficiency that reflects 21st-century demands of college and career readiness.
- Features a variety of instructional strategies, including cooperative and contextual learning.
- Recognizes the influence of adults' experiences and backgrounds on their learning.
- Is provided by instructors with expertise in mathematics content and the pedagogical knowledge to teach it to adults.
- Includes ongoing support for adult mathematics instructors to develop their expertise.

Guidelines for Adult Mathematics Instruction

Analysis of the NMAP report recommendations determined that 18 of the 45 recommendations were relevant to adult mathematics instruction. These recommendations and the guidelines derived from them fall into three broad areas: mathematics content, instructional strategies, and teacher preparation. Below are the guidelines related to each area. The report includes a discussion of the guidelines in each area, including relevant research.

Mathematics Content

Concepts of mathematical proficiency widely accepted in K–12 education are applicable to adult learning. The relative emphasis placed on the various content areas, topics, and levels of abstraction should be informed by adult students' skills and goals.

1. Mathematics content should emphasize a consistent link between math concepts learned and their use in context and form a coherent progression of learning.
2. The content topics of fractions, decimals, percent, and reasoning with proportions are essential and should form the instructional core for adult mathematics education.

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3. Algebraic thinking is essential for decision making in daily life and the workplace. Elements of algebra, therefore, should be introduced early to all students in adult mathematics instruction.
4. All content strands (number, geometry and measurement, algebra, and statistics) should be included in varying degrees at all levels of adult mathematics instruction.

Instructional Strategies

Adults bring with them experiences, both positive and negative, that influence their learning. Mathematics instruction should attend to these experiences and proceed in a practical, coherent, and integrated way, with careful monitoring of student progress.

1. Adult students should be able to demonstrate all aspects of mathematical proficiency: conceptual understanding, procedural fluency, strategic competency, and adaptive reasoning. Through their learning experiences, they should be developing a productive disposition toward learning and using mathematics.
2. Computational fluency requires not only proficiency with arithmetic and algebraic procedures, but also understanding of why and how they work. Both aspects should be part of adult mathematics instruction.
3. Adult mathematics instruction should recognize and address negative affective factors, both beliefs and emotions that can interfere with learning.
4. Adults' goals and experiences offer opportunities to embed instruction in meaningful contexts. Instruction should include connections to student interests, work situations, and everyday life (e.g., following recipes, basic accounting required on the job or at home) to stimulate engagement and promote applicability.
5. Formative assessment reveals student thinking and monitors progress and should be common practice in adult education.
6. A variety of student grouping strategies should be implemented to enhance learning through communication and collaboration. Those with a clear structure are most likely to succeed.
7. Mathematics instruction should include the technology used in the contexts for which students are preparing.

EXECUTIVE SUMMARY***Teacher Preparation***

Teacher preparation for adult mathematics instruction must be sufficiently intensive and focused on providing instructors with a strong base of mathematics content and pedagogical knowledge.

1. Qualifications for teaching math to adults should include a strong background in mathematics, an understanding and appreciation of the need for a broad conception of mathematical proficiency, and knowledge of the diverse range of performance expectations associated with adults' different mathematics learning goals.
2. Mathematics teachers in adult education need pedagogical knowledge that enables them to analyze student work to determine depth of understanding and implement appropriate instructional strategies.
3. In-service professional development must be of an intensity and quality to ensure acquisition of the necessary mathematics content and pedagogical knowledge and skills.²
4. In addition to professional development for the current workforce, other alternatives should be considered, such as using mathematics specialists, changing hiring practices to recruit more teachers with a background and experience in mathematics teaching, or seeking innovative preservice and early-service teacher preparation practices.

Designed to be a blueprint for future work by policymakers, administrators, and researchers in the field of adult education, these guidelines outline the mathematics content adults need to know, strategies for teaching adults this content, and the preparation of adult education instructors who teach mathematics. Additional research, however, is needed for a better understanding of how mathematics instruction can best be provided for different populations of adults pursuing diverse goals. More information about the relationships among teacher characteristics and preparation, instructional strategies, and student outcomes would be useful, as would an assessment of the effectiveness of current in-service professional development programs.

² As described in Sherman et al., (2007), Porter, Garet, Desimone, & Birman, (2004), and Smith, Hofer, Gillespie, Solomon, & Rowe (2003).

Introduction

Many adults need to improve their basic mathematics skills for success in postsecondary education and employment, according to several large-scale national and international surveys (U.S. Department of Education, 2003; Organisation for Economic Co-Operation and Development [OECD], 2005; Tamassia et al., 2007).³ Results from The Adult Literacy and Lifeskills Survey (OECD 2005), which examined adults' literacy and numeracy skills in the context of daily life and work across seven countries, including the United States, showed that those with low numeracy skill levels are more likely to be unemployed for six months longer than those at higher levels and three times more likely to receive social assistance payments. Moreover, acquiring mathematics skills is an obstacle for many adults. In 2009, passing rates on the GED mathematics exam were the lowest among the five academic subjects tested (mathematics, science, reading, writing, and social studies) (American Council on Education, 2010).

Mathematics instruction for adults needs improvement to enable adults to build their mathematics skills so that they can succeed in the workforce, advance in their careers, and participate fully as citizens (General Educational Development [GED] Testing Service, 2010; Kutner, Greenberg, & Baer, 2005; Tamassia et al., 2007). Those who teach mathematics in adult education often lack the appropriate credentials and expertise, and many willingly admit their discomfort in offering mathematics instruction (Gal & Schuh, 1994; Mullinix, 1994; Smith et al., 2003; Ward, 2000).

To address these needs, MPR Associates, Inc., and a team of adult numeracy experts from the Center for Literacy Studies at the University of Tennessee, Rutgers University, and TERC developed guidelines for adult numeracy instruction.⁴ The team analyzed *Foundations for Success: The National Mathematics Advisory Panel Final Report* (the NMAP report) to determine its applicability to adult mathematics instruction, reviewed relevant research literature, and developed guidelines that were submitted to four subject-matter experts (SMEs)⁵ for review and approval. This report describes these efforts.

³ The National Assessment of Adult Literacy (U.S. Department of Education, 2003) found that 93 million adults in the United States possess basic or below-basic skills in prose, document, and quantitative literacy. Some 19,000 adults in the U.S. participated in the NAAL, representing the entire population aged 16 and older. Twenty-two percent of adults performed at the below-basic level and 33 percent at the basic level.

⁴ This work was supported by the Office of Adult and Vocational Education (OVAE), U.S. Department of Education.

⁵ The subject-matter experts are: Dr. Daniel Berch (University of Virginia). Dr. Francis (Skip) Fennell (McDaniel College), Dr. Russell Gersten (Instructional Research Group), and Dr. Michael McCloskey (The Johns Hopkins University). Drs. Berch, Fennell, and Gersten served on the NMAP. Their biographies are in [Appendix B](#).

INTRODUCTION

Intended for policymakers, administrators, and researchers in the field of adult education, this report offers guidelines for (1) the mathematics adults need to know in the contexts of managing daily life, succeeding in the workforce, preparing for the GED test, and enrolling in postsecondary education; (2) strategies for teaching adults the mathematics they need within these contexts; and (3) teacher preparation for adult education instructors who teach mathematics.

Report Development

This report is based on the following activities:⁶

- Development of a process and criteria for (1) analyzing the NMAP report and determining its relevance to adult education and (2) establishing standards of evidence for studies and other materials consulted for the literature review.
- Analysis by the project team of recommendations in the NMAP report to determine their relevance to adult mathematics education.
- Oversight and guidance from four subject matter experts (SMEs) regarding the NMAP report analysis and its potential relevance to the development of guidelines for adult mathematics instruction.
- A review of extant literature and other relevant materials on adult education, adult mathematics instruction, and numeracy education.
- The report also draws heavily on the widely accepted definition of mathematical proficiency set forth in the National Research Council report *Adding It Up* (2001), described in the textbox on the following page, and noted in the NMAP report.

Report Organization

The report has three main sections. Each section begins with a brief overview, offers guidelines for adult mathematics instruction, discusses relevant recommendations from the NMAP report, and supports the guidelines with pertinent material from the NMAP report and research. Following the discussion of each guideline, implications for adult education are described.

- The **mathematics content** section provides definitions of mathematical proficiency and then describes the mathematics content adults need to know for success within the different contexts of their lives and goals.

⁶ A more thorough discussion of how the report was prepared is in [Appendix A](#).

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- The **instructional strategies** section describes ways to teach mathematics to adults, outlining instructional methods considered effective for developing mathematical proficiency while responding to adults' individual needs.
- The **teacher preparation** section addresses the required mathematics content expertise and pedagogical knowledge and skills needed by adult mathematics instructors and suggests potential approaches to teacher preparation and professional development.

Definitions:

MATHEMATICS SKILLS FOR ADULTS

Various terms are used in this report to describe adults' mathematics skills, including mathematical literacy, adult numeracy, and quantitative literacy. The terms have similar definitions, and they often are used interchangeably. For example, the Organisation for Economic Co-Operation and Development (OECD) defines mathematical literacy as:

An individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgments and to use and engage with mathematics in ways that meet the needs of the individual's life as a constructive, concerned and reflective citizen (OECD, 2005, p. 1).

In the Adult Literacy and Lifeskills Survey, adult numeracy is described in terms of five levels that range from understanding and completing basic numerical tasks, including those requiring such one-step operations as counting or sorting, to understanding "complex representations and abstract and formal mathematical and statistical ideas" (OECD, 2005, p. 17). The National Assessment of Adult Literacy defines quantitative literacy as the knowledge and skills required to identify and perform computations on numbers found in print material (Kutner et al., 2005, p. 2).

MATHEMATICAL PROFICIENCY

According to the National Research Council (2001, p. 116), mathematical proficiency consists of the following five strands:

- Conceptual understanding: comprehension of mathematical concepts, operations, and relations.
- Procedural fluency: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately.
- Strategic competence: ability to formulate, represent, and solve mathematical problems.
- Adaptive reasoning: capacity for logical thought, reflection, explanation, and justification.
- Productive disposition: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

All of these terms, depending on the research being cited and the context, are used in this report to refer to the basic mathematics skills and knowledge adults need for success in postsecondary education, entry-level employment, and everyday life.

Mathematics Content

This section addresses the question: What mathematics content should be taught in adult education? Drawing on NMAP Recommendations 1–4 and 6, as well as relevant research, four guidelines for the content of adult mathematics instruction were developed. Because adults generally enter adult education with specific goals, the mathematical demands of these goals are also discussed, as they may influence the selection and sequence of content to be offered in adult math instruction.

Adults' goals for their participation in adult education (Tamassia et al., 2007) may include one or more of the following:

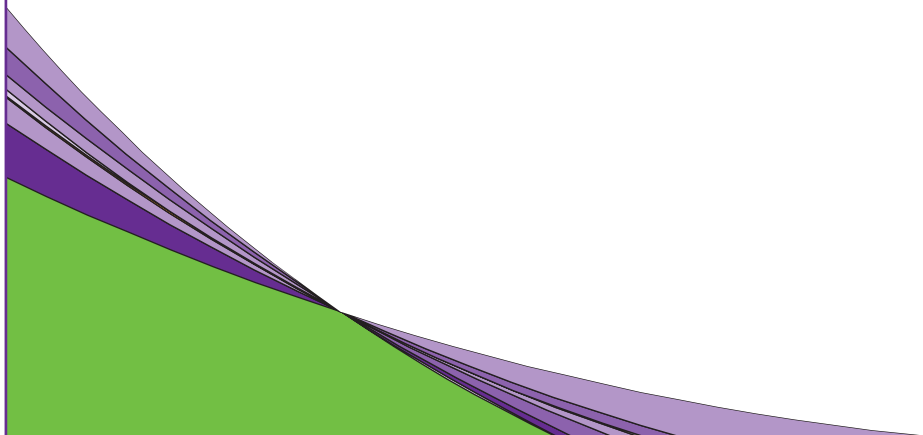
- Managing everyday life tasks, including budgeting, helping with children's homework, and contributing to community activities.
- Entering or advancing in the workforce, including preparation for entry-level positions and qualification for mid-level positions or technical/credentialed positions.
- Obtaining a GED or another secondary school credential.
- Preparing for further education in a certificate or academic degree program.

The mathematical demands associated with these goals are recognized in the following guidelines and discussed in more detail below. The four guidelines focus on specific content areas and emphasize the need for a coherent progression of learning that blends utility and abstraction to provide adults with a solid foundation of mathematical knowledge.

Guidelines

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- 1** Mathematics content should emphasize a consistent link between math concepts learned and their use in context and form a coherent progression of learning.
- 2** The topics of fractions, decimals, percent, and reasoning with proportions are essential and should form the instructional foundation for adult mathematics education.
- 3** Algebraic thinking is essential for decision making in daily life and the workplace. Elements of algebra, therefore, should be introduced early to all students in adult mathematics instruction.
- 4** All content strands (number, geometry and measurement, algebra, and statistics) should be included in varying degrees at all levels of adult mathematics instruction.



NMAP Recommendations

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RECOMMENDATION 1

A focused, coherent progression of mathematics learning, with an emphasis on proficiency with key topics, should become the norm in elementary and middle school mathematics curricula. Any approach that continually revisits topics year after year without closure is to be avoided.

By the term focused, the Panel means that curriculum must include (and engage with adequate depth) the most important topics underlying success in school algebra. By the term coherent, the Panel means that the curriculum is marked by effective, logical progressions from earlier, less sophisticated topics into later, more sophisticated ones. Improvements like those suggested in this report promise immediate positive results with minimal additional cost.

By the term proficiency, the Panel means that students should understand key concepts, achieve automaticity as appropriate (e.g., with addition and related subtraction facts), develop flexible, accurate, and automatic execution of the standard algorithms, and use these competencies to solve problems.

RECOMMENDATION 2

To clarify instructional needs in Grades PreK–8 and to sharpen future discussion about the role of school algebra in the overall mathematics curriculum, the Panel developed a clear concept of school algebra via its list of Major Topics of School Algebra (Table 1, p. 16).

School algebra is a term chosen to encompass the full body of algebraic material that the Panel expects to be covered through high school, regardless of its organization into courses and levels. The Panel expects students to be able to proceed successfully at least through the content of Algebra II.

RECOMMENDATION 3

The Major Topics of School Algebra should be the focus for school algebra standards in curriculum frameworks, algebra courses, textbooks for algebra, and end-of-course assessments.

RECOMMENDATION 4

A major goal for K–8 mathematics education should be proficiency with fractions (including decimals, percent, and negative fractions), for such proficiency is foundational for algebra and, at the present time, seems to be severely underdeveloped. Proficiency with whole numbers is a necessary precursor for the study of fractions, as are aspects of measurement and geometry. These three areas—whole numbers, fractions, and particular aspects of geometry and measurement—are the Critical Foundations of Algebra. The Critical Foundations are not meant to comprise a complete mathematics curriculum leading to algebra; however, they deserve primary attention and ample time in any mathematics curriculum.

RECOMMENDATION 6

All school districts should ensure that all prepared students have access to an authentic algebra course—and should prepare more students than at present to enroll in such a course by Grade 8. The word *authentic* is used here as a descriptor of a course that addresses algebra consistently with the Major Topics of School Algebra (Table 1, p. 16). Students must be prepared with the mathematical prerequisites for this course according to the Critical Foundations of Algebra (p. 17) and the Benchmarks for the Critical Foundations (Table 2, p. 20).

Source: U.S. Department of Education (2008). *Foundations for Success: The National Mathematics Advisory Panel Final Report*.

Mathematics Instruction and the Goals of Adult Learners

Below are descriptions of four contexts for adult mathematics content—everyday life, the workplace, the GED mathematics test, and readiness for community college—and the nature of mathematical demands and proficiency associated with them.

Mathematical Demands of Everyday Life

Adults routinely use mathematics—and not just simple arithmetic—in their daily lives, Gal (2000) illustrates this by identifying three common types of numeracy situations:

1. Counting, quantifying, computing, or manipulating numbers and generating responses with clear right or wrong answers;
2. Making sense of verbal or text-based messages based on quantitative data, but not manipulating numbers; and
3. Finding and considering multiple pieces of information to determine a course of action, often without clear correct answers.

Mathematics instruction for adults should prepare them to negotiate these types of situations successfully.

Mathematics instruction for adults also should take account of how adults actually use math in daily life, as well as previous math instruction that may have incorporated alternative algorithms and strategies. Some standard computational procedures learned in school may not be the most useful for many everyday situations. For example, shoppers often estimate, choosing to sacrifice precision or accuracy to save time or lighten the mental load. Alternatively, adults may calculate accurately in a flexible but non-standard way that fits the particular numbers involved (Lave, 1988). Masingila, Davidenko, & Prus-Wisniowska (1996) described similar findings in examining the situational dimension of mathematics performance in the workplace. Efficient estimation procedures, therefore, would seem to be a key objective for adult education in mathematics.

Another term used to describe the mathematics that adults use daily is “quantitative literacy,” which involves gaining command of both the skills needed to search out quantitative information and the ability to analyze and apply it in making decisions (Madison, 2006, p. 2323). Madison suggests that—for everyday purposes—relevant mathematics includes arithmetic, proportional reasoning, and measurement, but this

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mathematics also can be applied in sophisticated contexts such as estimating health risks or economic rates of change. Over the years, the demands involved in understanding everyday phenomena are increasingly related to mathematics and technology, as evident in the fuller description of quantitative literacy that appears in [Appendix E](#). Mathematics instruction for adults, then, while providing foundational knowledge and appropriate computational skills, also should enable adults to use math flexibly in various situations.

Mathematical Demands of the Workplace

Some researchers have suggested that success in the workplace requires mathematical knowledge and problem-solving abilities broader than those fostered by instruction that emphasizes procedures. For example, a wide range of knowledge and skills is necessary to accomplish such practical tasks as allocating resources, scheduling, understanding the role of quantitative information in the operation of systems, and using technological tools to quantify or display quantitative information (U.S. Department of Labor [SCANS], 1992; Mayer, 1992; Packer, 1997; Forman & Steen, 1999).

Researchers studying workplace demands have identified some of the common mathematical content topics involved, while emphasizing that mathematics for work centers on problem solving that is deeply embedded in the situation (Packer, 2003; Marr & Hagston, 2007; Forman & Steen, 1999; Wedege, 2000; Hoyles, Wolf, Molyneux-Hodgson, & Kent, 2002).⁷ Packer (2003) argues that the goals and content of mathematics courses should focus, in part, on problems that “American workers get paid to solve, those that American citizens should have informed opinions about, or those that American consumers actually need to solve” (p. 35). He adds the topic of “trade-offs” to his list, indicating that problems are contextual and the answer to a complex problem is not always found by using mathematics alone.

Studies of workers in many settings, ranging from banks to construction, have produced similar findings.⁸ Across multiple workplace environments, commonly used skills include algebraic thinking, estimation, judging the necessary degree of accuracy, logic, and managing and interpreting data. Workers also must be able to communicate mathematically and solve problems with numerical components (FitzSimons, 2005). Marr & Hagston (2007) found that across industries and even at the entry level, workers were asked to measure, calculate, solve proportional problems, use formulae, and understand the implications of the data they were collecting and entering.

⁷ A sample of these topics appears in [Appendix D](#).

⁸ See, for example, studies of carpet layers (Masingila, 1994; Masingila et al., 1996); designers, nurses, bank employees (Hoyles, Noss, & Pozzi, 1999); chemical sprayers (FitzSimons & Mlcek, 2004); carpenters (Millroy, 1992); pool builders (Zevenbergen & Zevenbergen, 2009); and auto workers (Smith, 2002).

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Instructional content related to adults' workplace goals, then, should include both proportional reasoning and algebraic thinking. Further, solving problems at work depends to a large extent on the work context and requires a broader approach than do the mathematics problems sometimes taught in school (Masingila, 1994).

Mathematical Demands of the GED Test

Passing the GED test is an important goal for many adult education students (General Educational Development Testing Service, 2010).⁹ Content for the GED mathematics test reflects four major strands of mathematics, giving approximately equal representation to each, and emphasizing skills and understanding also important outside the academic setting. Each test form is constructed so that 20 percent of the items are procedural, 30 percent are conceptual, and 50 percent involve application, modeling, or problem solving. The following summary was derived from an analysis of these content specifications and the official practice tests published for the field:

- For the *Number, Number Sense, and Operations* strand, rational number equivalents, including proportions and percents, are embedded in problem situations to be modeled mathematically. Test-takers must carry out calculations with pencil and paper for half of the test, but calculators are allowed for the other half.
- For the *Measurement and Geometry* strand, test-takers are asked to model and solve problems involving perimeter, area, and volume of figures. The use of perpendicularity, parallelism, congruence, and similarity in geometric figures and the Pythagorean Theorem are included. On a coordinate plane, they must find, use, and interpret the slope and y-intercept of a line, as well as apply transformations (translations, rotations, reflections, and dilations.)
- For the *Data, Statistics, and Probability* strand, test-takers are required to analyze and interpret data organized in tables, charts and graphs, and frequency distributions. This includes finding measures of central tendency and dispersion, evaluating inferences and arguments, and considering the difference between correlation and causation. Test-takers also are required to determine the probability of occurrence of an event after listing all possible outcomes.
- For the *Algebra, Functions, and Patterns* strand, test-takers are asked to model situations in linear, quadratic, rational, or exponential functions by analyzing and/or producing verbal descriptions, tables, graphs, and equations. Algebraic techniques

⁹ For information about the GED testing program and test specification, see *Technical Manual: 2002 Series GED Tests*, http://www.acenet.edu/Content/NavigationMenu/ged/pubs/TechnicalManual_2002SeriesGEDTests.pdf.

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of simplifying expressions, evaluating formulas, and solving equations also are assessed individually and in combination.

While the GED mathematics test requires increased proficiency with abstract symbolic mathematical content, it maintains a strong connection to the more practical skills needed for goals related to everyday life and the workplace.¹⁰

Mathematical Demands of Community College Readiness

Two-year community and technical colleges, herein called simply “community colleges,” offer the most popular options for adult GED recipients who pursue further education. Early results from an ongoing study show that 42.9 percent of those passing the GED tests enroll in postsecondary education, and 77.8 percent of those are enrolled in an institution offering programs of two or fewer years in length (Patterson, Zhang, Song, & Guison-Dowdy, 2010). Community colleges offer two distinct programs that include different math courses, different entry requirements, and, therefore, different pre-college preparation:

- *Certification programs* (short courses of study leading to certificates of achievement in various career fields) often do not require college-level mathematics courses, but instead offer courses on mathematics topics related to the specific occupation. For example, at Truckee Meadows Community College in Reno, NV, students in the automotive technician program must take “Math for Technicians,” and those in the culinary arts program must take a quantitative reasoning course, “The Business Chef” (<http://www.tmcc.edu/catalog/1011/worksheets>). The equivalent of a pre-algebra course is required for entry into the programs.
- *Associate’s degree programs* (A.A. or A.S., with credits transferable to a four-year institution) require a college-level mathematics course. Community colleges use placement tests to determine whether a student is prepared for college-level mathematics; those judged underprepared are often assigned to developmental (remedial) math courses (see [Appendix E](#) for a description of developmental math course content).

The level of mathematics content required for a given course is indicated by the prerequisite for entry; only pre-algebra is required for mathematics courses in many certification programs. To prepare students for employment in specific occupational areas, instruction focuses on practical applications of arithmetic and pre-algebra topics. For example, the course description for “The Business Chef” advises students to

¹⁰ The type of content for the GED mathematics test described above will prevail until a new version of the test is introduced as part of the GED Testing Service’s 21st Century Initiative. At that time, the content is expected to be aligned with the Common Core State Standards for College and Career Readiness (CCSSO 2010).

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carry a calculator and lists topics such as organizing food, labor, and overhead costs; purchasing; scheduling; completing spreadsheets; and calculating break-even points, topics similar to those for mathematics for the workplace described in [Appendix D](#) (<http://www.tmcc.edu/catalog/1011/courses/index.php?DescSection=C#CUL>).

The nature of the mathematics proficiency required in a college-level mathematics course differs considerably from that required to pass the GED or to succeed in a certification program. The demands can be compared by examining the specifications of the GED mathematics test and common college placement tests (CPTs), such as the College Board ACCUPLACER® or ACT COMPASS® (see [Appendix F](#)). With respect to content, CPTs cover a wider range of algebra topics than does the present GED test, in which the demands do not extend to intermediate algebra.

The current (2002) GED mathematics test emphasizes practical aspects of number, data, algebra, and geometry, and the required problem solving is often situated within adult contexts. CPTs, in contrast, tend to assess abstract reasoning in arithmetic and algebra. This is evident in the published sample items from each assessment. Table 1 (see below) shows a striking difference between the percentage of purely symbolic items found in the CPT samples and the percentage found in a published GED practice test.

Table 1. Symbolic Test Items: College Placement Tests vs. the GED		
Test	Section	Percentage of symbolic items
ACCUPLACER	Arithmetic	80.00%
	Elementary algebra	90.00%
	College mathematics	90.00%
COMPASS	Arithmetic/pre-algebra	43.00%
	Algebra	81.25%
GED	Entire practice test (form PF)	16.00%

Sources: College Board (2007). *ACCUPLACER® Sample Questions for Students*. New York: Author, pp. 4–8 (<http://professionals.collegeboard.com/profdownload/accuplacer-sample-questions-for-students.pdf>).

American College Testing (2004) *COMPASS/ESL Sample Test Questions: A Guide for Students and Parents, Mathematics*. Iowa City: ACT, Inc. (<http://www.act.org/compass/sample/pdf/numerical.pdf>).

Content Guidelines Discussion

Guideline 1: A Focused, Coherent Progression of Mathematics Content Related to Adult Learning Goals, Blending Practicality with Abstract Mathematical Concepts and Ideas

NMAP Recommendations 1–4 and 6 highlight the importance of developing proficiency that includes conceptual understanding, computational facility, and problem solving and a logical progression of learning that includes computational procedures, understanding of the mathematical ideas underlying the procedures, and numerous opportunities to solve applied problems using the mathematics being taught.¹¹ The recommendations also list topics under three specific content areas (whole numbers, fractions, geometry and measurement) recommended as the focus of curricula, instruction, and assessments in K–8 mathematics, so that students are prepared for success in an authentic algebra course.

The list of Critical Foundations (see Table 2 below, on p. 14) provides a platform of core arithmetic procedures from which math proficiency related to all adult goals can develop. As a list of essential topics, the table provides a valuable baseline from which to consider what proficiency in mathematics means for adults. The grade-level benchmarks, while not literally applicable to adult education, indicate a coherent pathway of instruction.

Because adults have diverse goals, mathematical proficiency for adult students may not be captured best by a single list of competencies. The requirements for proficiency involve additional factors not mentioned in the NMAP Critical Foundations list.

In particular, available research suggests that a variety of practical applications, though beneficial for young learners, are absolutely critical for adult learners' goals. The same mathematics content can span a continuum from the practical to the abstract, as adults' goals vary from preparation for everyday life and the workforce to the GED test and college-level mathematics courses. Figure 1 (see below, on p. 15) illustrates Guideline 1, showing how similar mathematical content (integers) might take different forms and demand different mathematical competencies from adults in each context.

¹¹ The NMAP report endorses the definition of proficiency offered by the National Research Council in *Adding It Up* (National Research Council, 2001, p. 116). See textbox on p. 3.

MATHEMATICS CONTENT**Table 2. Foundations for Success
Benchmarks for the Critical Foundations****Fluency With Whole Numbers**

1. By the end of Grade 3, students should be proficient with the addition and subtraction of whole numbers.
2. By the end of Grade 5, students should be proficient with multiplication and division of whole numbers.

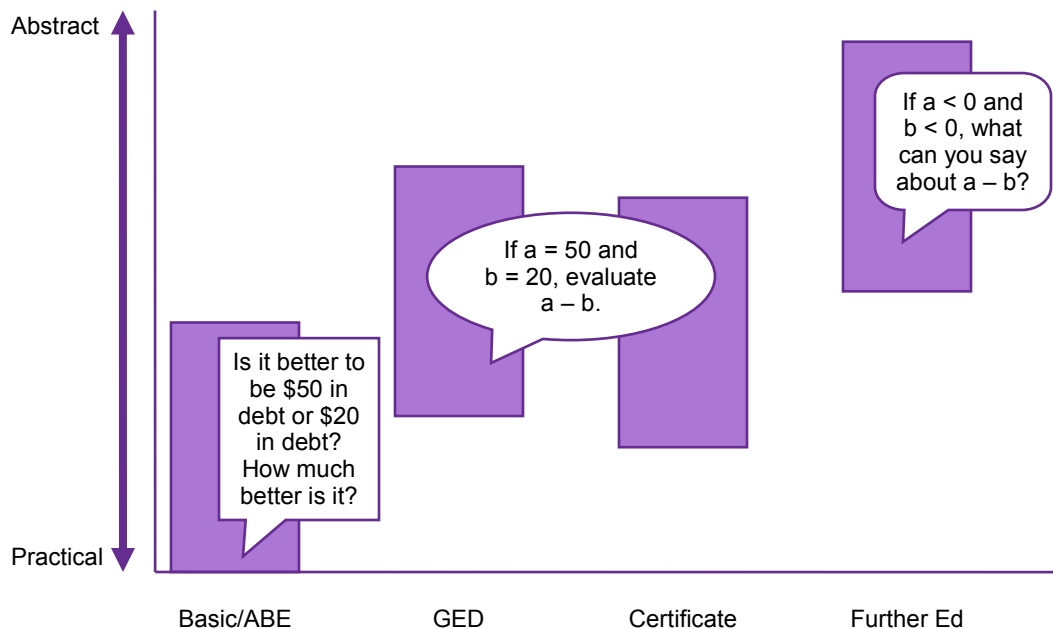
Fluency With Fractions

1. By the end of Grade 4, students should be able to identify and represent fractions and decimals, and compare them on a number line or with other common representations of fractions and decimals.
2. By the end of Grade 5, students should be proficient with comparing fractions and decimals and common percents, and with the addition and subtraction of fractions and decimals.
3. By the end of Grade 6, students should be proficient with multiplication and division of fractions and decimals.
4. By the end of Grade 6, students should be proficient with all operations involving positive and negative integers.
5. By the end of Grade 7, students should be proficient with all operations involving positive and negative fractions.
6. By the end of Grade 7, students should be able to solve problems involving percents, ratios, and rates and extend this work to proportionality.

Geometry and Measurement

1. By the end of Grade 5, students should be able to solve problems involving perimeter and area of triangles and all quadrilaterals having at least one pair of parallel sides (i.e., trapezoids).
2. By the end of Grade 6, students should be able to analyze the properties of two-dimensional shapes and solve problems involving perimeter and area, and analyze the properties of three-dimensional shapes and solve problems involving surface area and volume.
3. By the end of Grade 7, students should be familiar with the relationship between similar triangles and the concept of the slope of a line.

Source: U.S. Department of Education (2008). *Foundations for Success: The National Mathematics Advisory Panel Final Report*, p. 20.

MATHEMATICS CONTENT**Figure 1. Levels of Mathematical Abstraction Required for Adult Goals: Integer Examples¹²****Implications for Adult Education: Guideline 1¹³**

Based on the preceding analysis, adult mathematics teachers should choose mathematical content based on the contexts for which students are preparing. Instruction should explicitly include foundational topics students need for full understanding of the mathematics they will use. Teachers should have an accurate view of the extent to which practical applications and abstract/symbolic understanding are required so that they can achieve an appropriate balance in the classroom.

Guideline 2: The Instructional Foundation for All Mathematical Goals: Fractions, Decimals, Percents, and Reasoning with Proportions

NMAP Recommendation 4 urges development of proficiency with fractions (including decimals, percents, and negative fractions) for K–8 students, especially as a foundation for studying formal algebra. Recommendation 12 re-emphasizes this point, noting that difficulty with rational numbers (which include fractions, decimals, and

¹² Figure 1 was derived from the NMAP report and the literature review.

¹³ Throughout this report, Implications for Adult Education are based on the analysis of the NMAP report and research cited.

MATHEMATICS CONTENT

percents) is a major obstacle to progress in mathematics. These recommendations apply to adults as well as children.

There are additional reasons beyond preparation for algebra that adult education should focus attention on the concept of rational numbers. As indicated in the previous discussion of adult goals, adults need to be proficient with fractions, decimals, and percents to thrive in all aspects of adult life. As consumers, adults daily confront situations demanding a clear understanding of percents, and, as workers, they may need to use fractions to measure precisely or compare probabilities. As GED candidates, adults must solve problems requiring proportional reasoning, and, as college students, they need to master fractions before moving on to more advanced mathematical topics.

Implications for Adult Education: Guideline 2

According to the preceding analysis, understanding rational numbers should be at the forefront of adult mathematics instruction for all adult learners. Learners need to attain a deep understanding of the rational number system and the relationships among fractions, decimals, and percents. This level of understanding can only develop with extensive use of rational numbers in class, including computation practice and opportunities to solve problems and explain solutions. This foundational knowledge is needed to support mathematics competency in all goals.

It may not, however, be realistic or even desirable for adults to focus on these foundational topics only in anticipation of a formal algebra course. Programmatic time constraints, as well as those of adult students, may indicate the need for a shorter instructional route requiring that some aspects of the content take priority over others, while still emphasizing the development of understanding and reasoning. For example, when studying the addition and subtraction of fractions, it is important for all learners to understand the need for common denominators and how to find them. Learners whose goals require one or two formal algebra courses may benefit from learning a technique for finding the common denominator that involves breaking the existing denominators down into their prime factors and then putting factors together to create the least common denominator. This may serve as a model for an abstract process with variables.

For students whose goals do not require formal algebra, instruction can be limited to common fractions encountered in measurement (halves, fourths, eighths, etc.) and other daily activities (thirds, fifths, tenths). Here, the process of finding common denominators can be achieved more simply by merely reasoning about the particular fractions to find a number divisible by both existing denominators. By using the simpler fractions and a common-sense process, the instructor can employ physical

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representations or drawings that connect directly to student experience, an approach well suited for that level of learning. Extensive practice manipulating fractions that require the more formal prime factoring approach can use up limited learning time without providing practical or conceptual benefit.¹⁴

Guideline 3: Algebraic Thinking for Daily Life and the Workplace—Introducing Elements of Algebra Early to All Students

NMAP Recommendations 2 and 3 emphasize the central importance of algebra and set the expectation that K–8 students should be able to “proceed successfully at least through the content of Algebra II” (p. xvii). Recommendation 6 refers to the topic list for an *authentic* algebra course.¹⁵ These recommendations apply to adults as well, but need to be modified as appropriate for each adult goal (Manly & Ginsburg, 2010). Including all content through algebra II exceeds the preparation demands for most adult goals, except for those seeking to bypass developmental mathematics in entering a community college academic program.

The *NMAP Major Topics of School Algebra and Adult Goals Crosswalk* (see Table 3 below, on p. 18), compares the NMAP authentic algebra topic list with the mathematical demands of adult goals. The crosswalk shows that several algebraic topics are useful for solving problems and interpreting phenomena in everyday life and in the workplace.

In addition to the algebraic topics in Table 3, other important elements of algebraic thinking can be developed early without using formal symbolic algebra. Kieran (2004) lists some of these ways of thinking: “analyzing relationships between quantities, noticing structure, studying change, generalizing, problem solving, modeling, justifying, proving, and predicting” (p. 149).

Implications for Adult Education: Guideline 3

The preceding discussion suggests that adult mathematics instruction should anticipate algebra at all levels of instruction. This means not only developing foundational topics, but also incorporating elements of algebraic thinking throughout. Several researchers have suggested that elements of algebraic thinking introduced early can be

¹⁴ In the same way that differentiated instruction aims to meet the learning needs of different students, instruction can be modified to meet the various goals of adult learners.

¹⁵ “Authentic” is used in the NMAP report to characterize “an algebra course that addresses algebra consistently with the Major Topics of School Algebra” (Table 1, p. 16; NMAP, 2008, p. xviii).

MATHEMATICS CONTENT**Table 3. NMAP Report Major Topics of School Algebra and Adult Goals Crosswalk**

The Major Topics of School Algebra (<i>NMAP report</i> , p. 16)	Everyday Life	Workplace Entry	GED	Community College Certificate Program	Community College Associate Degree
Symbols and Expressions					
Polynomial expressions			X	X	X
Rational expressions	P	P	P	P	X
Arithmetic and finite geometric series	P	P	X	X	X
Linear Equations					
Real numbers as points on the number line	X	X	X	X	X
Linear equations and their graphs	P	P	X	X	X
Solving problems with linear equations	X	X	X	X	X
Linear inequalities and their graphs	P	P	X	X	X
Graphing and solving systems of simultaneous linear equations		P	X	P	X
Quadratic Equations					
Factors and factoring of quadratic polynomials with integer coefficients			X		X
Completing the square in quadratic expressions					X
Quadratic formula and factoring of general quadratic polynomials					X
Using the quadratic formula to solve equations					X
Functions					
Linear functions	X	X	X	X	X
Quadratic functions—word problems involving quadratic functions	P	P	X		X
Graphs of quadratic functions and completing the square			X		X
Polynomial functions (including graphs of basic functions)			P	P	X
Simple nonlinear functions (e.g., square and cube root functions; absolute value; rational functions; step functions)	P (step functions)	P (step functions)	X	P (step functions)	X
Rational exponents, radical expressions, and exponential functions	P (exp. functions)	P (exp. functions)	X	P (exp. functions)	X
Logarithmic functions					X
Trigonometric functions		P	P		
Fitting simple mathematical models to data	P	P	X	P	X
Algebra of Polynomials					
Roots and factorization of polynomials			X		X
Complex numbers and operations					X
Fundamental theorem of algebra					X
Binomial coefficients (and Pascal's Triangle)					X
Mathematical induction and the binomial theorem					X
Combinatorics and Finite Probability					
Combinations and permutations, as applications of the binomial theorem and Pascal's Triangle	P	P	X		X
SUGGESTED ADDITIONS					
Technology					
Calculators	X "Four-function"	X Scientific	X Scientific	X Scientific	X Graphing
Spreadsheets	X	X	X	X	X

NOTE: X = requires formal, in-depth study of the topic.

P = Does not require formal study, but could involve selected representations of the topic, particularly as used for practical purposes.

MATHEMATICS CONTENT

instrumental in showing students that mathematics, including algebra, can be meaningful and useful for all goals (Kaput, Carraher, & Blanton, 2007; Manly & Ginsburg, 2010). Further, these researchers have noted that algebraic thinking offers an advantage for successful functioning in everyday life, the workplace, and occupational certification programs. Algebraic thinking can help provide a deeper, connected understanding of arithmetic procedures and estimation techniques and is fundamental in recognizing patterns and making generalizations about recurring phenomena. Further, it is essential for writing expressions and equations used in spreadsheets that describe the mathematical relationships in the workplace.

Guideline 4: Including All Content Strands (Number, Geometry and Measurement, Algebra, and Statistics) in Varying Degrees at All Levels of Adult Mathematics Instruction

The NMAP was charged with suggesting the optimum preparation in K–8 education for the study of algebra and does not make recommendations for a curriculum integrating topics from different strands of mathematics. Two documents issued by the National Council of Teachers of Mathematics, however, *Principles and Standards for School Mathematics* (NCTM, 2000) and *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence* (NCTM, 2006) may provide some useful guidance. NCTM has suggested that mathematical content from all content strands (number and operations, algebra, measurement and geometry, and data analysis and probability) be addressed, with varying emphasis, at all grade levels within K–12 education.

For example, in the NCTM *Focal Points*, algebra is represented in Grade 1 as identifying, describing, and applying number patterns, while developing strategies for basic facts and data analysis is represented in Grade 3 as constructing and analyzing various graphs. The Common Core State Standards for Mathematics (<http://www.corestandards.org/the-standards/mathematics>) include algebraic thinking, geometry and measurement, and data as early as kindergarten, and the *Equipped for the Future* (EFF) standards¹⁶ incorporate similar recommendations for adults at all proficiency levels of their Performance Continuum (National Institute for Literacy, 2004).

¹⁶ The National Institute for Literacy developed the *Equipped for the Future* standards to identify what adults need to know and be able to do to carry out their roles and responsibilities as workers, parents, family members, and citizens. The standards address: communication (reading, writing, listening, speaking); decision making (mathematics, problem solving, planning); interpersonal skills; and lifelong learning skills.

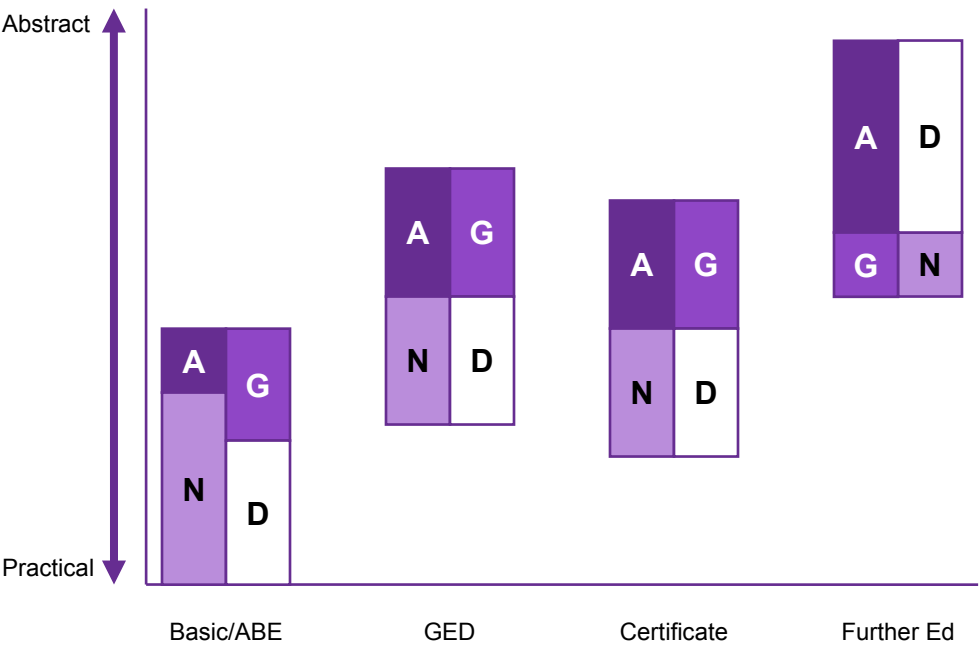
Implications for Adult Education: Guideline 4

According to the preceding analysis, because adult students may have been exposed to some foundational content during prior schooling and in daily life, it seems appropriate for adult educators to integrate topics from various content strands during instruction, to fill the gaps among the isolated pieces of mathematical knowledge that students may remember.

The NMAP was tasked with focusing on preparation for algebra, and their recommendations point to that goal. The NMAP report does not make recommendations about the practice of developing algebraic reasoning before formal algebra instruction, nor does the report mention data and statistics. Since adult education is charged with preparing students for a variety of goals, instruction must include content from all math content strands that play important roles in the workplace and daily life or appear on the GED test.

Figure 2 (see below) illustrates Guideline 4 by showing the relative emphasis of the different content strands at all levels along the practical to abstract continuum shown in Figure 1 (p. 15).

Figure 2. Representation of Content Areas in a Goal-Based Continuum of Mathematical Abstraction¹⁷



“A,” “G,” “N,” and “D” represent mathematics content strands: algebra, geometry, number, and data.

¹⁷ Figure 2 was derived from the NMAP report and the literature review.

Summary

The NMAP recommendations for content list topics under whole numbers, fractions, and geometry and measurement and emphasize a coherent progression of learning to prepare K–8 students for success in authentic algebra courses. The same topics describe a core of arithmetic procedures applicable to adult education.

Based on the analysis of NMAP recommendations and research presented here, these guidelines recognize that adults come to adult education with various goals, each requiring an approach to learning that fits with the mathematical demands of that goal. Everyday life and the workplace demand a practical approach focused on real-world problem solving. Passing the GED test and going on to earn an associate's degree demand more attention to the abstract nature of mathematics. A blend of utility and abstraction consonant with adult goals can promote success for adults in mathematics.

According to the research previously cited and the project's subject matter experts, mathematics instruction for adults should include a strong emphasis on rational numbers as the foundation for further study and for applying mathematics to life situations. This is true even when students are not yet proficient with all operations involving whole numbers.

Instruction also should recognize the importance of studying algebra for many different adult goals and the need to include elements of algebraic thinking for all of them. Simultaneous development of topics from the major strands of mathematics (number, algebra, geometry and measurement, and data and statistics) at all levels of instruction should be the norm for promoting a broad understanding of mathematics and its applications.

The following section addresses pedagogy, that is, how the mathematical content described in this section should be taught to adults, with a special focus on developing mathematical proficiency that includes more than simply knowing computational procedures.

Instructional Strategies

The preceding section described the mathematics content deemed important for adults to know to achieve their various goals. This section addresses how best to teach that content to adults, as well as some barriers that can interfere with the instructional process. Instructional strategies that can be effective in helping adults attain the mathematical proficiency necessary for achieving both immediate (e.g., managing everyday life, getting a job) and long-range goals (e.g., career advancement, postsecondary education) also are described. NMAP Recommendations 10–14, 25–26, and 38 were judged relevant to pedagogy for adult math instruction. These NMAP recommendations and relevant research are reflected in the following guidelines.

The NMAP recommendations relevant to pedagogy highlight the importance of computational fluency, conceptual understanding, contextual learning, formative assessment, and social and affective factors that influence learning. They also take note of the report of the NMAP Task Group on Instructional Practices, which states:

There is no one ideal approach to teaching mathematics; the students, the mathematical goals, the teacher's background and strengths, and the instructional context all matter. With regard to children, findings suggest that it is especially important to:

- Monitor what students understand and are able to do mathematically using brief but valid and reliable tests;
- Design instruction that responds to students' strengths and weaknesses, based on research when it is available; and
- Employ instructional approaches and tools that are best suited to the mathematical goals, recognizing that a deliberate and conscious mix of strategies will be needed (NMAP, 2008, p. xxiv).

Guidelines

INSTRUCTIONAL STRATEGIES

- 1 Adult students should be able to demonstrate all aspects of mathematical proficiency: conceptual understanding, procedural fluency, strategic competence, and adaptive reasoning. Through their learning experiences, they also should be developing a productive disposition toward learning and using mathematics.¹⁸
- 2 Computational fluency requires not only knowledge of efficient procedures, but also understanding of why they work. Both aspects should be part of adult mathematics instruction.
- 3 Adult mathematics instruction should recognize and address negative affective factors, including both beliefs and emotions that can interfere with learning.
- 4 Adults' goals and experiences offer opportunities to embed instruction in meaningful contexts. Instruction should include connections to student interests, work situations, and everyday life (e.g., following recipes, basic accounting required on the job or at home) to stimulate engagement and promote applicability.
- 5 Formative assessment exposes student thinking and monitors progress and should be common practice in adult education.
- 6 A variety of student grouping formats should be implemented to enhance learning through communication and collaboration.
- 7 Mathematics instruction should include the technology used in the contexts for which students are preparing.

¹⁸ For definitions of all aspects of mathematical proficiency, see the Textbox, Mathematical Proficiency Defined on p. 3.

NMAP Recommendations

INSTRUCTIONAL STRATEGIES

RECOMMENDATION 10

To prepare students for Algebra, the curriculum must simultaneously develop conceptual understanding, computational fluency and problem-solving skills. Debates regarding the relative importance of the aspects of mathematical knowledge are misguided. These capabilities are mutually supportive, each facilitating learning of the others. Teachers should emphasize these interrelations; taken together, conceptual understanding of mathematical operations, fluent execution of procedures and fast access to number combinations jointly support effective and efficient problem solving.

RECOMMENDATION 11

Computational efficiency with whole number operations is dependent on sufficient and appropriate practice to develop automatic recall of addition and related subtraction facts, and of multiplication and related division facts. It also requires fluency with the standard algorithms for addition, subtraction, multiplication, and division. Additionally, it requires a solid understanding of core concepts, such as the commutative, distributive, and associative properties. Although the learning of concepts and algorithms reinforce one another, each is also dependent on different types of experiences, including practice.

RECOMMENDATION 12

As with learning whole numbers, a conceptual understanding of fractions and decimals and the operational procedures for using them are mutually reinforcing. One key mechanism linking conceptual and procedural knowledge is the ability to represent fractions on a number line. The curriculum should afford sufficient time on task to ensure acquisition of conceptual and procedural knowledge of fractions and of proportional reasoning. Instruction focusing on conceptual knowledge of fractions is likely to have the broadest and largest impact on problem-solving performance when it is directed toward the accurate solution of specific problems.

RECOMMENDATION 13

Mathematics performance and learning of groups that have traditionally been underrepresented in mathematics fields can be improved by interventions that address social, affective, and motivational factors. Recent research documents that social and intellectual support from peers and teachers is associated with higher mathematics performance for all students, and that such support is especially important for many African-American and Hispanic students. There is an urgent need to conduct experimental evaluations of the effectiveness of support-focused interventions both small- and large-scale, because they are promising means for reducing the mathematics achievement gaps that are prevalent in U.S. society.

NMAP Recommendations

INSTRUCTIONAL STRATEGIES—CONTINUED

RECOMMENDATION 14

Children's goals and beliefs about learning are related to their mathematics performance. Experimental studies have demonstrated that changing children's beliefs from a focus on ability to a focus on effort increases their engagement in mathematics learning, which in turn improves mathematics outcomes: When children believe that their efforts to learn make them "smarter," they show greater persistence in mathematics learning. Related research demonstrates that the engagement and sense of efficacy of African-American and Hispanic students in mathematical learning contexts not only tends to be lower than that of white and Asian students but also that it can be significantly increased.

RECOMMENDATION 25

Teachers' regular use of formative assessment improves their students' learning, especially if teachers have additional guidance on using the assessment to design and to individualize instruction. Although research to date has only involved one type of formative assessment (that based on items sampled from the major curriculum objectives for the year, based on state standards), the results are sufficiently promising that the Panel recommends regular use of formative assessment for students in the elementary grades.

RECOMMENDATION 26

The use of "real-world" contexts to introduce mathematical ideas has been advocated, with the term "real world" being used in varied ways. A synthesis of findings from a small number of high-quality studies indicates that if mathematical ideas are taught using "real-world" contexts, then students' performance on assessments involving similar "real-world" problems is improved. However, performance on assessments more focused on other aspects of mathematics learning, such as computation, simple word problems, and equation solving, is not improved.

RECOMMENDATION 38

Calculators should not be used on test items designed to assess computational facility.

Source: U.S. Department of Education (2008). *Foundations for Success: The National Mathematics Advisory Panel Final Report*.

Instructional Strategies Guidelines Discussion

Guideline 1: Mathematical Proficiency

A clear definition of mathematical proficiency is an essential framework for any discussion about mathematics instruction. The NMAP report recognizes the five strands of mathematical proficiency advocated by The National Research Council (2001): conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. (See Textbox Mathematical Proficiency Defined on p. 3 for a fuller definition.)

This definition of mathematical proficiency also has been adopted in other pivotal K–12 mathematics education publications, such as the National Council of Teachers of Mathematics (NCTM) *Focus in High School Mathematics* document (2009) and the Common Core State Standards issued by the Council of Chief State School Officers and National Governors Association (2010).

Ginsburg et al. (2006) applied the five strands of proficiency to adult numeracy, suggesting that along with content and context, they offer a meaningful framework for describing productive adult numeracy. The three components (proficiency, content, and context) served as the basis for their analysis of existing adult education state standards in the United States; national standards documents from other English-speaking countries; and the specifications for numeracy assessments designed for adults.¹⁹

Implications for Adult Education: Guideline 1

The preceding analysis indicates that teachers should design instruction so that each strand of mathematical proficiency is addressed as students work with a topic (Ginsburg et al., 2006). The first step toward that goal may be as simple as using graphic organizers or asking questions to stimulate thinking, such as the following:

- How would you organize what you know and what you need to know?
- What would the problem look like in a picture?
- How did you know that...?

¹⁹ These assessments include the General Educational Development test (GED), Test of Adult Basic Education (TABE), Comprehensive Student Assessment Systems (CASAS), National Assessment of Adult Literacy (NAAL), and Adult Literacy and Lifeskills Survey (ALL).

INSTRUCTIONAL STRATEGIES

- What is another way that you could have found the answer?
- How would you have found the answer if you forgot the formula?
- Explain why...
- How does this relate to what we did last week?

Specific aspects of mathematical proficiency are discussed in the following sections.

Guideline 2. Computational Fluency

The NMAP report discusses computational fluency in Recommendations 10–12, which stress the development of automatic recall of arithmetic facts and algorithms, as well as the supportive nature of conceptual understanding when the two are developed simultaneously. Similarly, concurrent instruction in both computational fluency and conceptual understanding is recommended for adult mathematics instruction.

The adult education literature, however, suggests a need to go beyond fluency with standard algorithms. For example, many students in adult education are immigrants who can comfortably and effectively use math algorithms that differ from those commonly used in the United States. The algorithms are efficient, accurate, and sensible to the user (Ciancone, 1996; Schmitt, 2006; Zaslavsky, 1973; Tamassia et al., 2007), but can seem mysterious and even confusing to others. When students can use such algorithms successfully, there is little reason to discourage their use just because they may be non-standard or unfamiliar to the teacher.

Recognizing variation among individuals in mathematical practices and procedures can provide a basis for further instruction, such as discussions about why and how different procedures work. Caution must be exercised, however, because some practices may be confusing and not necessarily beneficial to further mathematics learning. These practices might include informal, self-developed math procedures that are effective in certain situations, but not generalizable or easily transferred to other contexts (Carraher, 1991; D'Ambrosio, 1997; Knijnik, 1996; Powell & Frankenstein, 1997).

Besides knowing the basic arithmetic facts and effective procedures for carrying out computations, other aspects of computational proficiency can be advantageous for adults. Having a well-developed “operational sense” for deciding which procedures should be applied is particularly important for practical problem solving in real situations (and for word problems on tests). Operational sense also involves knowing what kind of answer to expect. For example, a conceptual understanding of what it means to multiply positive or negative integers or rational numbers less or greater

INSTRUCTIONAL STRATEGIES

than one will help in problem solving, estimation, and checking for reasonableness. Knowing how to calculate, when to calculate, what calculation(s) to perform, and what level of accuracy is appropriate are goals for students at all levels. Instruction should address all of these aspects of computational fluency (Ginsburg, 2008; Reys, 1991; Rubenstein, 2001).

Implications for Adult Education: Guideline 2

According to the preceding analysis, practice with basic arithmetic facts and computational algorithms is an accepted method of instruction, but it should not be the only one used with adults. Adults who have tried for years but failed to master basic facts or remember a procedure can be good candidates for an alternative approach. Strategies that incorporate conceptual understanding can be effective here. For example, a student can learn to think of 8×7 as $(8 \times 5) + (8 \times 2)$ or $40 + 16$ to get the correct answer of 56. Relying on “easier” facts and processes of decomposing and combining numbers can facilitate learning in several ways. Mental mathematics becomes easier for the learner. By using such procedures, students develop insights into the commutative, associative, and distributive properties of multiplication and addition. Simultaneously, teachers should help adult students see the connections among the mathematical properties (distributive, associative, commutative) that provide the reasons for why the procedures work. Understanding these properties is of paramount importance for success in algebra.

While adults work toward the goal of automatic recall, a calculator can be a useful aid. The content lists for the various goals described in the previous section, however, show that relying solely on a calculator for fluency is not adequate. Mental math and estimation are important forms of computation for everyday life and the workplace. Success in both depends on recall of basic arithmetic facts and recognition of the appropriate operation to be used.

For practical situations, the appropriate level of precision must be determined. A real-life situation in which numbers are embedded may suggest the most appropriate computational technique for a good solution. An estimate may be good enough to answer the question or make the decision. The numbers involved may be conducive to finding the answer mentally. Written procedures may be necessary for an accurate answer in a simple situation, but a calculator may be better for more complex problems. The instructional goal is to enable students to be comfortable with many computational methods, so they can choose an efficient method and—perhaps—use another to check their answers.

Guideline 3. Affective Factors in Adult Learning

In Recommendations 13–14, the NMAP report recognizes that affective factors can influence an individual’s mathematics learning. These recommendations note that mathematics performance of underrepresented groups “can be improved by interventions that address social, affective, and motivational factors” (p. xix), which reduce students’ vulnerability to negative stereotypes about their mathematical ability. The report notes further that students who believed that math ability was innate (the “math gene”) did not persist in complex problem solving as long as did those who believed that their efforts made a difference. Available research literature on adults confirms that attitudes and beliefs can have a powerful impact on learning and therefore should be addressed by teachers.

Like children and adolescents, adults may believe that they don’t really need math or that math is not useful or connected to their goals. They also may believe that mathematical ability is innate, and that achievement is not connected to hard work (Evans, 2000; Wedege, 1999, 2002; Wedege & Evans, 2006). They may have negative perceptions of themselves as math learners, based on their own reactions or those of others, such as teachers and parents (Evans, 2000; Wedege & Evans, 2006). They also may not understand the relationship of their own efforts to their success (Nesbit, 1996).

Both emotions and beliefs can interfere with learning. Negative feelings about mathematics or one’s own ability to understand mathematics (math anxiety) affect students’ willingness to engage with mathematics and their ability to manage frustration when they encounter difficulty (NMAP, 2008; Jost, 1997; Cook, 1997; Evans, 2000).

The elements of past experience that give rise to math anxiety are discussed in the adult education literature. Singh (1993) found that adults’ math anxiety was caused by abstraction, perceived lack of relevance, and fear of failure induced by past instruction and testing. Wedege (2002) suggested that adults may have a narrow understanding of what doing mathematics is, seeing math learning as limited to memorizing a sequence of steps for a procedure that may or may not make sense to them. They may think that there is only one right way to solve a problem, and that finding the correct answer is the only purpose for doing mathematics.

Many adults also are affected by interrupted academic preparation. They may have left school because they were unsuccessful or because of events in their lives, or their opportunities to learn mathematics may have been limited, for example, by being tracked out of courses such as algebra (Jackson & Ginsburg, 2008).

INSTRUCTIONAL STRATEGIES**Implications for Adult Education: Guideline 3**

As the NMAP report and preceding analysis suggest, there is no one ideal way to approach teaching mathematics. The fact that adults come to mathematics instruction with varying levels of mathematical knowledge, both accurate and inaccurate, as well as attitudes, beliefs, and practices that may or may not be productive, presents a special challenge to educators. Strategies exist, however, that have been found effective by practitioners.

For example, according to Benseman (2001), successful adult numeracy teachers underscore the importance of attending to affective factors in ensuring students' commitment, engagement, and ultimate success. They help adults shift from a narrow view of mathematics learning as limited to mastering a set of rules and procedures to a view that sees mathematical activity as meaningful, flexible, productive, and relevant (Ginsburg et al., 2006). Adults also may have experiences or goals that can serve to motivate and ground further learning.

The next section discusses using contexts from everyday life and work to enhance learning.

Guideline 4. Meaningful Contexts

NMAP Recommendation 26 suggests that using “real-world” contexts can have positive effects for specific populations and on certain types of problem solving. In 2002, the U.S. Department of Education What Works Clearinghouse recognized functional-context education, which integrates job content with literacy skills, as an effective instructional approach (Fletcher, 2006). Students whose goals involve using mathematics in the real world are likely to benefit from instruction using context to introduce and explore concepts. This is relevant for many adult learners. Research on adult learning confirms that adults' real-world contexts do involve mathematical activity,²⁰ and adults likely would benefit from contextualized learning.

Research with adults also shows that using a range of contexts for teaching was most effective in improving trainees' abilities to generalize numeracy skills to new problems and situations (Wolf, Silver, & Kelson, 1990). A study from the United Kingdom found that more learners achieved numeracy qualifications when math instruction was embedded in adult vocational instruction than when it was taught as a separate course (Casey et al., 2007).

²⁰ See, for example, the studies of Gal, 2000; Gal, Van Groenestijn, Manly, Schmitt, & Tout, 2005; Coben, 2003; Hoyles et al., 1999; Rogoff, 1990; Lave & Wenger, 1991; Lave, 1991; Wenger, 1998; and Greeno, Eckert, Stucky, Sachs, & Wenger, 1999.

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A randomized controlled study of an intervention integrating mathematics instruction into high school career and technical education (CTE) programs resulted in improved student math performance on TerraNova and ACCUPLACER assessments (Stone, Alfeld, & Pearson, 2008). Instruction was designed to proceed incrementally, from being embedded within a single application to additional contextual applications and then to “traditional examples” likely to be encountered on standardized tests. This process of making the mathematics explicit and moving from context towards abstraction also was described in research by Boaler (1998, 2000).

Education researchers and practitioners agree that students’ active engagement with a problem is at the heart of what makes learning meaningful (Scholastic & Bill & Melinda Gates Foundation, 2010; Swain, Baker, Holder, Newmarch, & Coben, 2005). Contextualizing instruction is a strategy with the potential for engaging students, helping them to see the relevance and utility of mathematics and find meaning in their learning.

Implications for Adult Education: Guideline 4

According to the preceding analysis, contextualized learning, with its relevance to real-world issues, can provide strong motivation for adult students to develop strategies for solving problems, to reason within that context, and to learn and remember procedures, all aspects of mathematical proficiency. Drawing on the “funds of knowledge” that are part of adults’ homes and communities can enable teachers to help adults connect what they already know with academic knowledge (González, Moll, & Amanti, 2005). A contextual approach can connect the various strands of math content, offering the kind of coherence recommended by the *Principles and Standards for School Mathematics* (NCTM, 2000, p. 30). For example, a single project may begin with collecting measurements, providing numeric data that can serve as the basis for developing geometric principles and algebraic representations.

Incorporating context often means addressing mathematical concepts in a way that differs from a traditional sequence of topics. For example, situations involving shape or measurement offer a reason to use fractions in a meaningful way. Similarly, statistics about local issues can provide an opportunity for exploring methods of data collection and using multiple representations of rational numbers (e.g., $\frac{2}{5}$ of the residents vs. 42.1 percent of the residents).

Contextualized learning should be one of many learning methods used in the adult mathematics classroom. To be educationally effective—that is, more than mere training for a specific task—contextualized instruction should lead to a generalized abstraction of the mathematical concepts and an explicit connection to related concepts that have been studied (Stone, et al., 2008). Systematic instructional design is needed to

ensure that mathematical ideas coalesce into the deep understanding that equips students to handle new situations (Fuson, Kalchman, & Bransford, 2005).

Guideline 5. Formative Assessment

The NMAP report specifically recommends the use of formative assessment in the elementary grades (Recommendation 25), based on research studies that used assessment items from formal curriculum standards objectives to gauge student learning throughout the year. The report noted that “Formative assessment—the ongoing monitoring of student learning to inform instruction—is generally considered a hallmark of effective instruction in any discipline” (2008, p. 46). Often used in an informal way, it can help to shape instruction so that it attends to student needs in real time, before formal assessment takes place (Wiliam, Lee, Harrison, & Black, 2004).

The NMAP also viewed the process by which teachers gather information from informal observations as “promising.” Indeed, the NMAP report cites Freudenthal’s endorsement of immediate feedback: “It is more informative to observe a student during a mathematical activity than to grade his papers” (1973, p. 84).

Elementary and secondary teachers who participated in the Scholastic (2010) survey agreed with this view, saying that “formative, ongoing assessments during class, along with class participation and performance on class assignments, are the most important measures of student achievement” (p. 25). They reported using the data both to adjust instruction and to monitor class and individual progress. Garrison & Ehringhaus (n.d.) suggest that a balance of both formative and summative assessments is ideal for gathering the most accurate information about a student’s knowledge and understanding. Summative assessments tell where students stand with respect to standards at a given point in time, and formative assessments give immediate feedback about their learning to both students and teachers.

Involving students themselves in the assessment process also can be important for adults. Recognizing progress toward their goals is one of the factors that increases long-term persistence for adult students (Comings, 2007). In adult education, where individualized instruction is common practice, regular monitoring of student progress using written assessments is prevalent (Tamassia et al., 2007). It is unclear, however, whether these assessments are valid and reliable, nor whether they evaluate broad mathematical proficiency or merely narrow computational proficiency. Further, the research of Masingila et al. (1996) indicated that those who can answer multiple-choice questions featuring non-contextualized, purely symbolic manipulations may not be able to reason and solve problems confronting them outside school or in meeting their goals. Informal formative assessment, based on consciously

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gathering data from a variety of sources, including explanations of reasoning and problem-solving strategies, examples of representing and solving problems in multiple ways, and demonstrated computational skills, can offer valuable information for both teacher and student.

A review of the literature on the assessment of adult mathematics learning found examinations of summative (standardized) assessments administered to students upon entry into a program for diagnostic purposes and then at regular intervals to comply with the federal accountability requirements of the National Reporting System (AIR, 2006). Approved, commonly used assessments are the Test of Adult Basic Education (TABE), Comprehensive Adult Student Assessment Systems (CASAS), General Assessment of Instructional Needs (GAIN), and the Massachusetts Adult Proficiency Test. The American Institutes for Research *Review of the Literature in Adult Numeracy* (AIR, 2006) stated, however, that “neither assessment (TABE, CASAS) is adequate for conceptions of numeracy in the integrative phase, such as those embodied in many of the standards and frameworks influencing the field” (p. 44). This statement offers further evidence that information derived from these assessments is not sufficient to evaluate student progress on the broader goals of mathematical proficiency.

Implications for Adult Education: Guideline 5

The preceding analysis indicates that, as curriculum and assessment are being aligned to the broader goals of mathematical proficiency, instructors need to devise efficient means of formative assessment, so that students and teachers alike are aware of progress. For example, using observation and questioning as formative assessment techniques, teachers can determine how comfortable students are in using a number line to compare and order a collection of positive and negative fractions or decimals.

Questions like those suggested above to prompt reasoning and strategic competence also can be used to reveal gaps in student understanding that usually would not surface through completion of a practice worksheet of computational exercises. Teachers can modify instruction in a timely fashion to remedy the misunderstandings thus exposed before more consequential summative assessments are administered.

Guideline 6. Student Grouping

Collaborative or cooperative learning, used interchangeably here, refers to interactive instructional strategies ranging from formal structured approaches to the more casual “talk it over with your neighbor” approach. The NMAP report does not include a recommendation on cooperative learning, but some of the research examined for the report may be relevant. In the Learning Processes section, for example, the NMAP report notes evidence suggesting that collaborative learning can have “a positive in-

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fluence on mathematics performance and may be relatively important for minority students, particular those from low-income backgrounds,” but only if these situations have a clear structure. Citing a meta-analytic review of the effects of collaborative learning on math outcomes in elementary school, the report states that “overall, peer-assisted learning led to greater mathematics performance outcomes than did individual or competitively structured learning.” Effects varied, with larger effects found for minority students, those in urban settings, and those of low socioeconomic status (NMAP, 2008, p. 104).

Further, the NCTM process standards note that “communication is an essential part of mathematics and mathematics education” and that “conversations in which mathematical ideas are explored from multiple perspectives help the participants sharpen their thinking and make connections” (NCTM, 2000, p. 60).

In their meta-analysis of research on existing mathematics programs, Slavin and Lake (2008) found strong positive effects for highly structured cooperative learning within elementary math programs. Similarly, Slavin, Lake, & Groff (2009) found positive effects for middle and high school math programs using cooperative learning. In adult education, findings on student grouping and interaction similarly suggest that small-group instruction and a non-school environment increase adults’ confidence and success, particularly for traditionally underrepresented populations (King & Wright, 2003; Benseman & Tobias, 2003; DePree, 1998).

Adult education researchers point to specific factors making collaborative learning in adult education an especially compelling strategy. Using both language and mathematical skills, adult students participating in collaborative learning also are preparing for the requirements of the workplace. Hoyles et al. (2002, p. 5) write that the “techno-mathematical literacy” of the modern workplace demands “the ability to communicate mathematics to other users and interpret the mathematics used by others.” Further, the diverse cultural backgrounds of adult students create a unique opportunity for learning from each other. Not only do students clarify their own thinking by justifying it (Hatano & Inagaki, 1991), they also gain a better understanding by reconciling the different solution methods common in other countries. Taken as a whole, these studies suggest that collaborative learning is a promising strategy for strengthening adult mathematics instruction.

Implications for Adult Education: Guideline 6

According to the preceding analysis, well-designed cooperative learning activities allow teachers to create a balanced learning environment that combines the structure of being knowledge-centered with the flexibility to respond to learners and their backgrounds, goals, and needs. Adult education programs most often use individual

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lized or whole-class instruction, making less use of small groups or computer-assisted instruction (Tamassia et al., 2007).²¹ Programs often have open enrollment,²² allowing students to work at their own pace through lessons (in workbooks or online programs) based on their performance on standardized tests. Although this structure appears to be student-centered because of its focus on their knowledge level and goals, it does not force students to interact actively—either with subject matter or other students.

Adult education practitioners who use a variety of student grouping strategies report an increase in student confidence and collegial relationships within classes. Silver, Kilpatrick, & Schlesinger (1990) found that responsibility for one's own learning is fostered by the use of formative assessment and cooperative learning. Asking students to think aloud or explain their reasoning demands that they clarify and evaluate their own and others' understanding.

Guideline 7. Technology

The NMAP report concludes that, while technology-based tutorials and drill-and-practice software can be useful tools in improving student performance in specific mathematical areas, research is insufficient to identify the factors that make it effective (NMAP, 2008, p. xxiv). Technology, however, can be a tool for more general educational purposes, with the goal of engaging students interactively. Using the Internet and searching for information and applications can support adults' math learning, in or outside of formal classrooms (Pacuilla & Reder, 2008). Teachers can use technology for differentiated instruction (creating alternative instructional activities for students with different needs); to bring real-world situations into the classroom, creating contexts for mathematical exploration; and to simulate hypothetical situations.

Using calculators in mathematics classes is a well-debated topic among educators. The NMAP report notes that its review of related studies found “limited to no impact of calculators on calculation skills, problem solving, or conceptual development over periods of up to one year” (2008, p. xxiv). Recommendation 38 advises against

²¹ According to the Adult Education Program Survey (Tamassia et al., 2007), “46 percent of programs reported using individualized instruction a great deal (defined as more than 30 percent of total learner instruction time) and 43 percent of programs reported using classroom style instruction a great deal. Thirty-nine percent of programs reported using small group instruction within a classroom and 44 percent reported using computer-assisted instruction for 10 to 30 percent of the instruction time” (p. 35).

²² According to the Adult Education Program Survey (Tamassia et al., 2007), “Open enrollment policies, which allow learners to begin and stop classes at any time, were common among adult education programs. Overall, 79 percent of programs used open enrollment. Of these programs, 70 percent indicated that open enrollment was used for more than 80 percent of their instructional services” (p. 20).

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the use of calculators for test items designed to assess computational facility. Although allowed for some assessments adults encounter, including a section of the current GED math test, calculators are not permitted with other assessments, such as current versions of frequently used CPTs.

Because the NMAP report focused on the mathematics needed to be successful in algebra, it did not address the essential role played by technology in many real-life settings for which adult learners are preparing. Although adults need to be able to compute by hand when necessary, they should also feel comfortable using a calculator or other technological tools when appropriate (U.S. Department of Labor, 1992; Packer, 1997; Mayer, 1992; Forman & Steen, 1999).

Implications for Adult Education: Guideline 7

The preceding analysis suggests that a good reason to use calculators in adult education is that they are common in most workplaces and ubiquitous in households and for personal use (Hoyles et al., 2002; Glass & Wallace, 2001; AMATYC, 2002). Making calculators and related technological tools available to adults during classroom problem-solving activities gives them experience working with these tools as they would be expected to use them outside of school.

Similarly, spreadsheets are an important tool for collecting, representing, displaying, managing, and exploring mathematical information. Adults are likely to encounter these in the workplace. Asking adult students to reason about and create a “formula” for spreadsheet use can help them see the relevance and value of generalization, clarify the sometimes poorly understood notion of “variable,” and acquaint them with a tool they likely will be expected to use or interpret (Manly & Ginsburg, 2010).

Summary

The preceding analysis and available evidence indicate that instruction that attends simultaneously to all aspects of mathematical proficiency provides the strongest foundation for continued learning. Time should be spent on developing meaningful conceptual understanding, on understanding relationships among different representations of mathematical ideas, on flexibly moving from one representation to another, on establishing computational fluency, on developing a repertoire of problem-solving strategies, and on reinforcing the expectation that mathematics can and should make sense and that every student can learn to understand and use it. These aspects of mathematical proficiency are more likely to have a long-term impact on adults’ successful mastery of mathematics than lists of steps to be memorized or computational strategies.

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Instruction that attends to learners' backgrounds and learning histories as well as their goals also should incorporate strategies for formative assessment and collaborative learning. Learning to use technological tools such as calculators and spreadsheets as they are used in everyday life, in the workplace, and on assessments such as the GED, should be an integral part of instruction. These tools do not diminish the mathematical proficiency expected, but rather enhance learners' preparation for meeting their goals.

The following section discusses how to draw upon both content and instructional strategies, along with other considerations, to prepare teachers to provide effective math instruction to adults.

Teacher Preparation

This section addresses how teachers should be prepared and supported to teach the necessary mathematics content effectively to adults, in ways consistent with the preceding guidelines for content and instructional strategies. Based on the relevant NMAP recommendations (17, 19, and 20–22) and the research literature, four guidelines are suggested for teacher preparation.

Teacher preparation for adult mathematics instruction must take account of several important characteristics of the current adult basic education (ABE) teaching force. The teaching force is largely part-time and/or volunteer. In 2008–09, the ABE teaching force consisted of 58 percent part-time paid employees; 13 percent full-time paid employees; and 29 percent volunteers (<http://wdcrobcolp01.ed.gov/CFAPPS/OVAE/NRS/reports/index.cfm>). According to Tamassia et al. (2007), most adult education programs reported that the minimum educational requirement for their full- and part-time employees was a bachelor's degree and sometimes K–12 certification. Finally, most adult numeracy teachers have had little training in mathematics or mathematics education.²³ While adult education instructors typically focus on literacy and language, they often find themselves teaching mathematics as well, whether or not they were prepared to do so (Gal & Schuh, 1994; Ward, 2000; Mullinix, 1994). For all these reasons, strengthening teacher preparation in mathematics for adult education instructors is urgent. The guidelines that follow, informed by the NMAP recommendations, emphasize knowledge of mathematics content and instructional strategy and adequate opportunities for professional growth.

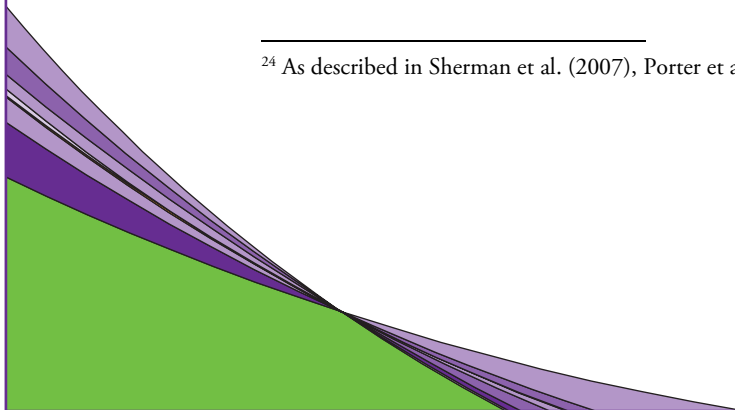
²³ Data on the percentage of ABE instructors who teach math are unavailable, but useful information comes from a large-scale national survey and two state-focused studies. In a survey of a nationally representative sample of 350 programs (serving 774,955 students per year), Gal & Schuh, (1994) found that, although more than 80 percent of adult students receive some math-related instruction, fewer than 5 percent of the teachers providing it were certified to teach mathematics. In a survey of 141 ABE math teachers in Massachusetts, “at least 55 percent re-ported having no training in mathematics pedagogy” (Mullinix, 1994). Finally, a study of Arkansas adult education teachers found that the typical GED instructor had an average of 4–6 years experience teaching GED mathematics; not only had they no formal math training beyond college algebra, but 64 percent also had not participated in any math training for the two years prior to the study (Ward, 2000).

Guidelines

TEACHER PREPARATION

- 1** Qualifications for teaching math to adults should include a strong background in mathematics, an understanding and appreciation of the need for a broad conception of mathematical proficiency, and knowledge of the diverse range of performance expectations associated with adults' different mathematics learning goals.
- 2** Mathematics teachers in adult education need pedagogical knowledge that enables them to analyze student work to determine depth of understanding and implement appropriate instructional strategies.
- 3** In-service professional development must be of an intensity and quality to ensure acquisition of the necessary mathematics content and pedagogical knowledge and skills.²⁴
- 4** In addition to professional development for the current workforce, other alternatives should be considered, such as using math specialists, changing hiring practices to include more teachers with a background and experience in mathematics teaching, or seeking innovative preservice and early-service teacher preparation practices.

²⁴ As described in Sherman et al. (2007), Porter et al. (2004), and Smith et al. (2003).



NMAP Recommendations

TEACHER PREPARATION

RECOMMENDATION 20

In an attempt to improve mathematics learning at the elementary level, a number of school districts around the country are using “math specialist teachers” of three different types—math coaches (lead teachers), full-time elementary mathematics teachers, and pull-out teachers. However, the Panel found no high-quality research showing that the use of any of these types of math specialist teachers improves students’ learning.

The Panel recommends that research be conducted on the use of full-time mathematics teachers in elementary schools. These would be teachers with strong knowledge of mathematics who would teach mathematics full-time to several classrooms of students, rather than teaching many subjects to one class. This recommendation for research is based on the Panel’s findings about the importance of teacher mathematical knowledge. Deploying teachers who have specialized in elementary mathematics teaching could be a practical alternative to increasing all elementary teachers’ content knowledge (a problem of huge scale) by focusing the need for expertise on fewer teachers.

RECOMMENDATION 21

Schools and teacher education programs should develop or draw on a variety of carefully evaluated methods to attract and prepare teacher candidates who are mathematically knowledgeable and to equip them with the skills to help students learn mathematics.

RECOMMENDATION 22

Research on teacher incentives generally supports their effectiveness, although the quality of studies is mixed. Given the substantial number of unknowns, policy initiatives involving teacher incentives should be carefully evaluated.

Source: U.S. Department of Education (2008). *Foundations for Success: The National Mathematics Advisory Panel Final Report*.

Teacher Preparation Guidelines Discussion

Guideline 1: A Strong Math Background, Understanding of a Broad Definition of Math Proficiency, and Knowledge of Performance Expectations for Different Adult Goals

NMAP Recommendations 17 and 19 emphasize the importance of teachers' content knowledge and its relationship to student achievement. Some research literature also underscores this point. In the adult education literature, a British study of the relationship between teachers' mathematics knowledge and student achievement found that teachers' content knowledge, as measured by highest mathematics degree attained, is an important factor. This study found relationships among teachers' math knowledge, their math teaching experience, and adult students' numeracy achievement (Cara & de Coulon, 2008a). These same authors also reported, however, that British teachers with postgraduate degrees were not as effective, on average, with lower-level ABE classes as those with lesser mathematics credentials (Cara & de Coulon, 2008b). No similar studies have been conducted with U.S. adult education teachers.

In K–12 general and mathematics education, there is a considerable literature on the relationship between teacher quality and student achievement, in which teacher quality is defined as mastery of subject matter and content knowledge for teaching. Some studies have found that the effects of teacher quality on educational outcomes can be more important than student socioeconomic status, class size, or teacher salaries (Darling-Hammond, 2000).

The NMAP Task Group on Teachers and Teacher Education concluded that although teachers' knowledge of mathematics is a positive factor in students' achievement, evidence about the relationship of teachers' mathematics content knowledge and students' mathematics achievement remains "uneven and has been surprisingly difficult to produce" (NMAP, p. 37). The mixed research results suggest that math content knowledge (as demonstrated by teacher certification or mathematics degree) is not necessarily sufficient for effectively teaching elementary and middle school children or adults at basic education levels (Cara & de Coulon, 2008b).

The findings discussed above raise the question of whether or not teacher knowledge and expertise should differ according to student levels or goals. In other words, do teachers need different knowledge and preparation if they will be teaching adult

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students at the various NRS levels and if their students are studying to meet the mathematical demands of everyday life, the workplace, the GED test, or transition to college?

Although all teachers must know their subject matter well and also know how to teach it, requirements for teaching at each NRS level may differ. Teachers of basic-level students need to understand whole and rational numbers, how number and operation sense develops, and how to support that development. A mathematics major, for example, may have a good understanding of how, why, and when decimal division works, but little understanding of the cognitive pitfalls encountered by adult students in learning this. On the other hand, an adult education teacher formally trained as an elementary school teacher may know how to help children develop understanding of and skill with decimal division, but have little idea of when adults will apply this knowledge or how it conforms to the larger principles of mathematics (the multiplicative identity and inverse operations).

Implications for Adult Education: Guideline 1

Based on the analysis and research cited above, adult mathematics instructors need a solid understanding of mathematics content, a view of mathematical proficiency encompassing more than just computational facility, and the ability to adjust instruction to adult students' various goals and education levels. Content requirements may vary according to adults' various goals. GED teachers need to analyze official practice tests and specifications so that instruction focuses on the critical concepts tested. Teachers whose students' goals focus on work and careers need to know how mathematics is used in specific fields. Transition-to-college teachers must have a deep understanding of mathematics and know how to help students make meaning of the mathematics. All teacher preparation, however, should consider placing mathematics content, appropriate to the levels and goals of the students, at the center of the program.

To teach mathematics effectively to adults, teachers must have an understanding of mathematical proficiency that includes the characteristics cited by the National Research Council: conceptual understanding, adaptive reasoning, strategic competence, procedural fluency, and a productive disposition. NMAP Recommendation 10, discussed in the previous section, endorses this definition. Professional development initiatives for adult numeracy instructors should encourage instructors to embrace a pedagogy that enacts this multidimensional definition of mathematics proficiency, appropriate to the level and goals of students.

Guideline 2: Pedagogical Knowledge Enabling Teachers to Analyze Student Work and Implement Appropriate Instructional Strategies

Content knowledge in mathematics is necessary but not sufficient for effective instruction for adults. Teachers also need to know how to teach that content effectively to adults. Ball and her colleagues have highlighted the importance of “mathematical knowledge for teaching,” by which they mean the knowledge “needed to carry out the *work of teaching mathematics*” (Hill, Rowan, & Ball, 2005, p. 373). They refer to the combination of content and pedagogical knowledge in mathematics required for effective teaching in K–12 education. These authors and their colleagues developed direct measures of mathematical knowledge for teaching and, using these measures, repeatedly have found a correlation between student achievement and mathematical knowledge for teaching. Baumert et al. (2010) also found a positive effect of pedagogical content knowledge, separate from the effect of teachers’ mathematics content knowledge, on students’ learning gains at the secondary level.

Teachers’ beliefs about the nature of mathematical proficiency, about mathematics as a subject, and about how people learn mathematics influence their decisions about how to teach mathematics, which instructional materials they use, and how they assess learning. With respect to adult education, a qualitative study of eight ABE mathematics teachers and their students found that the teachers viewed mathematics as difficult, intrinsically uninteresting, limited to computation, requiring mastery of “tricks,” and best tackled by repeated individual practice (Nesbit, 1996). Moreover, their teaching practices were aligned with their beliefs, in that they promoted just one method of learning: learn a rule and then apply it repeatedly until it becomes automatic. They seldom checked students’ comprehension and offered few opportunities for student interaction and discussion. Professional development for adult mathematics instructors also may need to strive to overcome these perceptions and practices.

Implications for Adult Education: Guideline 2

The analysis and research cited above indicate that professional development for adult education teachers should create opportunities for teachers to share and develop a repertoire of effective pedagogical practices that support their own pedagogical content knowledge. Teachers should know how to probe and assess student work with the intent of understanding student reasoning and then determining appropriate next steps for teaching and learning. They should be able to understand affective and cognitive factors influencing learning and to encourage productive dispositions towards learning and using mathematics. Teachers should have a chance to experience a variety of collaborative instructional strategies and uses of technology.

Guideline 3: Professional Development Promoting the Acquisition of Mathematical Content and Pedagogical Knowledge and Skills

NMAP Recommendation 19 urges that teachers have “ample opportunities to learn mathematics for teaching,” including professional development programs. There appear to be no preservice programs, however, for preparing mathematics instructors for adults. Moreover, an environmental scan of professional development practices found no information about practices for certification in adult mathematics instruction (Sherman et al., 2007). Several states have certification requirements, but for ABE in general, rather than for a specific discipline, such as math or reading. Against this backdrop of minimal teacher participation in mathematics preservice and certification programs, in-service professional development in adult numeracy has become a focus for improving teacher quality.

The Sherman et al. (2007) environmental scan summarized current and recent U.S. professional development initiatives, identifying five key features of high-quality professional development for elementary and secondary mathematics and science teachers: duration, collective participation, coherence, content knowledge, and active learning. These features were found to be related to improvements in teachers’ knowledge and skills and changes in teaching practice (Porter et al., 2004). A study examining standards-based professional development programs found them to have a positive effect on instructional practice (McREL, 2005).

Several in-service professional development programs have focused on mathematics. The environmental scan identified 30 professional development programs focused on mathematics for ABE teachers that were implemented between 1996 and 2006. Twenty programs (14 state-level and 6 national-level) met criteria for being included in the study because they had such “essential features” as duration and attention to teacher content knowledge (Sherman et al., 2007).²⁵

In the general adult education literature, two studies, while not focused on math instructors per se, might inform the design of future professional development programs for adult numeracy instructors. Smith et al. (2003) examined how adult education teachers changed after participating in three models of professional development focused on student motivation and persistence: (1) a multisession workshop

²⁵ Of these 20 adult numeracy professional development initiatives, one initiative was deemed “promising,” *Teachers Investigating Adult Numeracy* (TIAN). TIAN was adopted with modifications and piloted as part of OVAE’s *Strengthening America’s Competitiveness Through Adult Math Instruction* project.

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(experiential, active learning activities); (2) a mentor teacher group (study circles, peer coaching, and observation); and (3) a practitioner research group (teachers investigate their classroom practice and collect and analyze data). The study found that the factors related to change were hours of professional development attended, quality of professional development, and collaborative participation. The specific model of professional development did not appear to be a factor.

To design effective professional development programs for adult numeracy teachers, several potential barriers may need to be addressed. Through interviews with 60 adult basic education decision makers from 10 states, Wilson and Corbett (2001) identified five factors that negatively influence participation in professional development:

- Distance (professional development may not be available locally and employers may not pay for travel);
- Time and financial constraints (part-time instructors may not be paid for extended professional development);
- Information gaps (information about professional development opportunities may be sporadic);
- Mismatch between goals of programs and professional development offerings; and
- Lack of opportunities for face-to-face interaction about their work, the type of professional development educators perceive as most beneficial.

Implications for Adult Education: Guideline 3

The analysis and research cited here suggest that, at present, in-service professional development is the primary mechanism for preparing adult numeracy instructors. Professional development programs, therefore, have the potential to contribute significantly to the quality of mathematics instruction in adult education. Such programs should provide ongoing opportunities for teachers to strengthen their content knowledge *and* expand their pedagogical repertoire. Policy makers and professional developers at the state, regional, and local levels need to determine how to minimize barriers to participation and maximize the quality of the offerings.

Guideline 4: Using Math Specialists, Developing New Hiring Practices, Seeking Innovative Practices for Teacher Preparation

NMAP Recommendations 20–22 focus on alternative ways to bring competent math instructors into classrooms through the use of math specialists, new hiring practices, and teacher incentives. All of these may be relevant, to some extent, to adult basic education (Cara & de Coulon, 2008a, 2008b). The NMAP report notes, however, a lack of research supporting the use of math specialists, with research on the effectiveness of teacher incentives, while generally supporting their effectiveness, being “mixed.”

Recommendation 20 states that “Deploying teachers who have specialized in elementary mathematics teaching could be a practical alternative to increasing all elementary teachers’ content knowledge (a problem of huge scale) by focusing the need for expertise on fewer teachers” (p. xxii). Because of the current lack of formal mathematics training in the adult education workforce, the problem of scale is also acute in adult education. A variety of approaches, therefore, should be considered for strengthening adult math instructors’ content knowledge and pedagogical skills, in addition to math specialists (Gal & Schuh, 1994; Mullinix, 1994; Ward, 2000).

Some potentially replicable practices exist, such as induction practices used at the City University of New York transitions-to-college program, in which new teachers audit and give feedback in a master teacher’s class for one semester before taking responsibility for classes of their own (Hinds, 2009), or the use of regional math teacher-facilitators who provide support for teachers from several programs, as in the Massachusetts SABES Teacher to Teacher (T2T) program (Donovan, 2007), and in the regional learning community model in some of the *Teachers Investigating Adult Numeracy (TIAN)* states, such as Arizona (Wilson & Morales, 2008).

Implications for Adult Education: Guideline 4

Based on the foregoing analysis and evidence, the conditions noted above create a challenging context for providing ABE teachers with the professional development needed to improve their ability to teach mathematics well to adults. Pursuing innovative solutions, such as using technology to overcome issues of distance and lack of opportunities to meet face-to-face, is one possible approach. Another is to develop local leadership (mathematics mentors and mathematics specialists). Programs also might consider offering incentives to teachers to update and enhance their math content and pedagogical knowledge regularly. For example, scholarships for teachers for

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further in-depth study of the mathematics they teach might be especially appropriate for those teaching transition-to-college classes.

At present, given that there is no preservice preparation for adult numeracy instructors, professional development focused on mathematics content and pedagogical knowledge is a major strategy for improving instruction. Such programs should have the characteristics of effective professional development identified in the research literature and be accessible to adult education teachers in a variety of employment circumstances.

Summary

According to the preceding analysis and available evidence, effective mathematics instruction for adults requires that teachers have (a) relevant content knowledge at a deep level, to be able to make connections across concepts; (b) the ability to use that knowledge in teaching their specific students, along with an understanding of student thinking and reasoning, to be able to identify and address gaps and misunderstandings; (c) the ability to make connections across contexts in which mathematics is embedded or applied, as well as to develop and discuss abstract representations; and (d) beliefs about mathematical activity and mathematics learning and teaching that promote the use of multiple instructional strategies and the ability to move among them as appropriate. All forms of teacher preparation for adult mathematics instructors should incorporate these considerations.

Further, programs to improve adult numeracy instruction should (a) create opportunities for teachers already in the adult education workforce (in-service), while investigating the feasibility of expanding pathways for preservice or new teachers; (b) be designed with attention to duration, collective participation, coherence, content knowledge, and active learning; (c) be standards-based; and (d) consider how to overcome the constraints of distance, schedules, and limited financial resources.

Conclusion

The guidelines for adult numeracy instruction offered here relied heavily on an analysis of the National Mathematics Advisory Panel report and the existing, though limited, research on adults. With respect to content, all adults need proficiency with rational numbers (fractions, decimals, and percents) and proportional reasoning. In addition, elements of algebra, data and statistics, and geometry and measurement should be included in varying degrees at all levels. Instruction must follow coherent progressions with relative emphasis appropriate to student goals.

Pedagogy should involve strategies for promoting reasoning, problem solving, and communicating, all critical in applying mathematics to real situations. Opportunities for structured cooperative learning should be integrated into instruction. The use of progress monitoring assessment systems that are valid, reliable, and assess all aspects of proficiency, along with occasional informal formative assessments, are recommended practices.

Recommended approaches to teacher preparation emphasize improving math content and pedagogical knowledge through systematic high-quality professional development, as well as investigating the possibilities of using math specialists, modifying recruiting practices, and offering incentives as ways of bringing high-quality math instruction to adult education.

Additional research would be informative, particularly in furthering understanding of how adult mathematics learning can be maximized for specific adult populations pursuing diverse goals. Greater understanding of the relationships among teacher characteristics and preparation, instructional strategies, and student outcomes also is needed. Finally, research is necessary to determine the effectiveness of particular professional development programs and other approaches (math specialists, teacher incentives) designed to ensure that adult education programs have teachers with a background and interest in mathematics teaching and learning.

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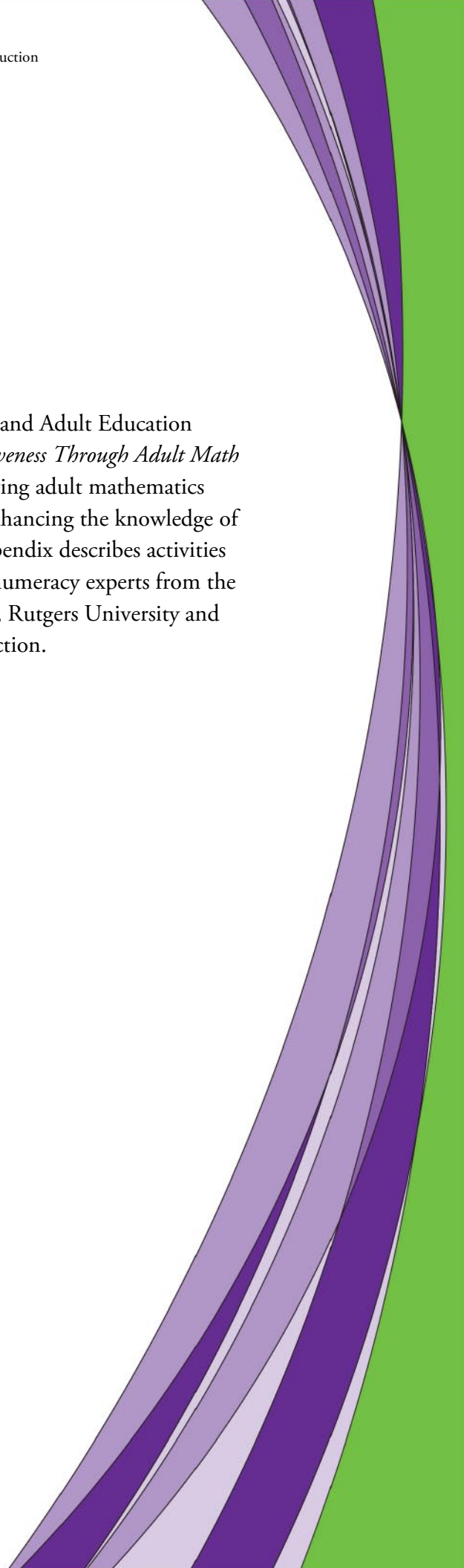
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Appendix A: Methodology

The U.S. Department of Education, Office of Vocational and Adult Education (OVAE), sponsored the *Strengthening America's Competitiveness Through Adult Math Instruction* project from 2007–11, with the goal of improving adult mathematics learning and instruction by strengthening the skills and enhancing the knowledge of those who teach mathematics in adult education. This appendix describes activities undertaken by MPR Associates, Inc. and a team of adult numeracy experts from the Center for Literacy Studies at the University of Tennessee, Rutgers University and TERC to develop guidelines for adult mathematics instruction.



Analyzing Foundations for Success: The National Mathematics Advisory Panel Final Report (NMAP)

The NMAP report was analyzed to determine which, if any, of its recommendations were applicable to adult education. Four subject matter experts (SMEs) with knowledge of mathematics and mathematics education served as consultants to the project, overseeing the analysis of the NMAP report and advising on the development of the guidelines for adult mathematics instruction. Three of the four experts also had participated on the NMAP, as noted below. The SMEs are:

- Dr. Daniel B. Berch, associate dean for research and faculty development, Curry School of Education, University of Virginia; member of the NMAP.
- Dr. Francis (Skip) Fennell, professor of education, McDaniel College; past president of the National Council of Teachers of Mathematics (NCTM); member of the NMAP and chair of the NMAP subgroup on conceptual knowledge and skills.
- Dr. Russell Gersten, executive director, Instructional Research Group; professor emeritus of educational research at University of Oregon; member of the NMAP and co-chair of the NMAP subgroup on instructional practice.
- Dr. Michael McCloskey, professor of cognitive science, Johns Hopkins University. Dr. McCloskey analyzed the results from the 2003 National Assessment of Adult Literacy (NAAL) quantitative tasks.

Complete biographies for the SMEs are included in [Appendix B](#).

The SMEs and project staff first reviewed components of the adult education system, characteristics of adult learners, adult students' goals and related mathematical demands of these goals, and current practices in adult mathematics instruction. The group also developed a work plan for examining the NMAP report.

The SMEs and project staff next developed criteria for analyzing the NMAP recommendations, determining that a recommendation would be judged "relevant" if it were pertinent to adult mathematics instruction and to the specific goals of adults, including navigating everyday life, obtaining a GED, transitioning to postsecondary education, and succeeding at work. The group further decided that the analysis should reflect issues specific to adults and adult education programs, such as cognitive processing in adults and opportunities for sustained instruction, as well as the characteristics of adults who enroll in adult education programs. Using these criteria,

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the SMEs and project staff each independently assessed the relevance of each recommendation in the NMAP report.

Individual assessments were reconciled to determine the overall applicability of the NMAP report recommendations to adult mathematics instruction. Based on a consensus of at least 80 percent agreement among the SMEs and project staff, the group identified 19 of the 45 NMAP recommendations as relevant to adult mathematics instruction and the goals of adult students. Not all of the 19 relevant recommendations relate directly to teaching adults mathematics, but all include components considered informative and worthy of attention in the discussion of adult mathematics instruction. The group agreed that the issues raised by the recommendations mattered more than their exact wording. “Relevant,” therefore, means that the issue addressed in the recommendation has implications for adult mathematics instruction, although the recommendation may not apply exactly as written to adult education.

The SMEs and project staff then refined the NMAP analysis by comparing their results with the *Synthesis of Relevant Extant Literature Reviews* completed by project staff. To complete the review, project staff consulted literature on adult education, adult mathematics education, and numeracy education to examine findings related to the goals and purposes of adult numeracy and identify potential guidelines for adult mathematics instruction. Findings from the summary are incorporated into this paper where appropriate.

Analyzing Other Relevant Research and Materials

The SMEs and project staff also established standards of evidence for other materials to be consulted for this report.¹ To be included in this report, a resource must reflect at least one of the following standards of evidence:

- Qualitative and quantitative studies with adults (i.e., convergent findings from multiple qualitative studies with adults; convergent findings from large-scale adult surveys and assessments);
- Qualitative and quantitative studies with related populations (i.e., relevant information that can be extrapolated from studies of K–12 students, traditional college-age students, and young English language learners);

¹ References in the bibliography are annotated with the appropriate levels of evidence (see Appendix C for a description of the levels of evidence).

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- Document analyses (i.e., derived from existing national or international policy or standards documents or assessment frameworks); and
- Recognized expertise (i.e., convergent positions emerging from publications of recognized experts; convergent professional judgment of recognized experts).

Based on these standards, the SMEs and project staff also considered the following research, materials, and sources of expertise:

- The Benchmarks derived from the NMAP report's Major Topics of School Algebra and Critical Foundations (pp. 16 and 20), examining them against the December 2009 draft of the Common Core State Standards in mathematics developed by the National Governors Association and Council of Chief State School Officers (<http://www.corestandards.org/>).
- Studies included in the *Synthesis of Extant Literature Reviews*.
- *Focus in High School Mathematics: Reasoning and Sense Making* by the National Council of Teachers of Mathematics and related topic books on Algebra, Data Analysis and Probability, and Geometry.
- Other studies conforming to the evidence criteria (included in the references).
- Using the analyses of the extant literature reviews, the NMAP Major Topics of School Algebra and Critical Foundations, and the 19 NMAP recommendations identified as relevant to adult mathematics instruction, the SMEs and project staff developed the guidelines for adult mathematics instruction.

Appendix B:

Subject Matter Expert Bios

Dr. Daniel B. Berch

Associate Dean for Research and Faculty Development
Curry School of Education, University of Virginia
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Dr. Daniel B. Berch currently serves as the Associate Dean for Research and Faculty Development at the Curry School of Education. Dr. Berch most recently served as Associate Chief of the Child Development and Behavior Branch at the National Institute of Child Health and Human Development, NIH, where he also directed the Program in Mathematics and Science Cognition and Learning. Dr. Berch came to the Washington, DC area in 1997 as an SRCD/AAAS Executive Branch Science Policy Fellow. He subsequently was appointed Senior Research Associate at the U. S. Department of Education, advising the Assistant Secretary for Educational Research and Improvement on technical and policy matters related to educational research. He has published a variety of articles on children's numerical cognition, mathematical learning disabilities, and spatial information processing. In addition, Dr. Berch served as an ex officio member of the U.S. Department of Education National Mathematics Advisory Panel.

APPENDIX B: SUBJECT MATTER EXPERT BIOS**Dr. Francis Fennell**

Professor of Education
McDaniel College
2 College Hill
Westminster, MD 21157

Dr. Francis Fennell is professor of education at McDaniel College and president of the National Council of Teachers of Mathematics. His research focuses on elementary and middle school math instruction. Dr. Fennell was named Case Professor of the Year for the state of Maryland. He has received multiple awards in the areas of mathematics and teacher education. Dr. Fennell also has played leadership roles within the Research Council for Mathematics Learning, Mathematical Sciences Education Board, National Science Foundation, Maryland Mathematics Commission, U. S. National Commission for Mathematics Instruction, and Association for Mathematics Teacher Educators. He currently serves on the National Mathematics Advisory Panel and has published in the areas of elementary and middle-grades mathematics education. Dr. Fennell holds a Ph.D. in mathematics education from the Pennsylvania State University.

Dr. Russell Gersten

Executive Director
Instructional Research Group
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Dr. Russell Gersten is executive director of the Instructional Research Group, a non-profit research institute, and a member of the National Mathematics Advisory Panel, co-chairing the subgroup on instructional practice. He also directs the mathematics component of the Center of Instruction, a comprehensive technical assistance center for No Child Left Behind that provides states and regional centers with research-based strategies for improving the quality of mathematics instruction. He is a nationally recognized expert in quantitative and qualitative research and evaluation methodologies. Dr. Gersten has conducted two syntheses of intervention research on teaching mathematics to low-achieving students and students with math disabilities. He served as an advisor for the mathematics component of the Title I evaluation in 2003, and he recently completed a research project on developing valid measures for early screening of students with mathematics disabilities. He also has conducted research on the use of technology to teach mathematics to students with disabilities. Dr. Gersten holds a Ph.D. in special education from the University of Oregon.

APPENDIX B: SUBJECT MATTER EXPERT BIOS**Dr. Michael McCloskey**

Professor, Cognitive Science

Department of Cognitive Science

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Dr. Michael McCloskey is professor of cognitive science at Johns Hopkins University. His research focuses on mental representation and computation, particularly in the areas of visual space cognition and lexical processing. He studies cognitive deficits in children and adults with brain damage or learning disabilities, normal cognitive representations and processes, and the disruptions of these when the brain is damaged or fails to develop normally. He has studied numerical cognition in normal adults, as well as numerical processing deficits in brain-damaged patients (acquired dyscalculia). Dr. McCloskey holds a Ph.D. in cognitive psychology from Princeton University.

Appendix C:

Levels of Evidence—

Adult Mathematics Instruction

Guidelines and Literature Review

1. Types of Resources Cited

Abbrev.	Description
QUANT/A	Quantitative study with adult population
QUAL/A	Qualitative study with adult population
QUANT/O	Quantitative study with related population (i.e., K–12, traditional-age college students, ELL children)
QUAL/O	Qualitative study with related population (i.e., K–12, traditional-age college students, ELL children)
DOC/A	Documents relating to adult populations, developed by federal or state agencies or professional organizations (i.e., standards documents, assessment frameworks)
DOC/O	Documents relating to populations other than adults, developed by federal or state agencies or professional organizations (i.e., standards documents, assessment frameworks)
EXPUB/A	Professional judgments by recognized experts about adults
EXPUB/O	Professional judgments by recognized experts about other populations
LITREV	Literature review

**APPENDIX C: LEVELS OF EVIDENCE—ADULT MATHEMATICS INSTRUCTION
GUIDELINES AND LITERATURE REVIEW**

2. Descriptors of Strength and Kinds of Evidence (listed from 1–4 in order of decreasing evidentiary strength)

1. Research studies with adults
 - Convergent findings from multiple qualitative studies with adults
 - Convergent findings from data from large-scale adult surveys and assessments
 - Convergent findings from multiple quantitative studies with adults
2. Research studies with related populations
 - Extrapolated from research with other populations (i.e., K–12, traditional-age college students, ELL children)
3. Document analyses
 - Derived from analyses of existing national or international policy or standards documents or assessment frameworks
4. Recognized expertise
 - Convergent professional judgments of recognized experts

Appendix D:

Mathematical Content Topics for the Workplace

Some researchers have suggested that success in the world of work requires broad mathematical knowledge and problem-solving skills, and they have identified specific mathematical content topics demanded by the workplace.

1. Packer describes the mathematics required to solve problems occurring in roles and competencies listed in the Secretary's Commission on Achieving Necessary Skills' (SCANS)¹ in the Content by Competency table shown on the following page (Packer, 2003, p. 41).
2. Marr & Hagston (2007) developed case studies of three worksites representing significant industries in Australia employing entry-level workers: residential healthcare, automotive parts production, and sheet metal products engineering. In each setting, employees and supervisors were interviewed and workers were shadowed as they completed their work tasks. The authors found the following mathematical activities prevalent across industries.
 - Measurement (including appropriate tape measures, rules, scales, gauges and dials, calipers, and other tools) was used daily, with the degree of accuracy dependent on the purpose. Estimation of measurements was also common, based on years of experience.
 - Calculations involving multiplication, addition, and subtraction of whole numbers and decimals were performed daily, but division was avoided. Percentages were calculated by many individuals. With a shift to the metric system, calculations with fractions were no longer used except "half" and "quarter," as related to time.

¹ These skills and competencies are described in the Secretary's Commission on Achieving Necessary Skills (1991). *What work requires of schools. A SCANS report for America 2000*. Washington, DC: U.S. Department of Labor.

APPENDIX D: MATHEMATICAL CONTENT TOPICS FOR THE WORKPLACE

Mathematics Required to Solve Frequently Occurring Problems in Four Roles and Five SCANS Competencies					
Problem Domains	Planning <ul style="list-style-type: none"> Budget Schedule Space Staff 	Systems and Processes <ul style="list-style-type: none"> Understand Monitor Design 	Interpersonal <ul style="list-style-type: none"> Negotiate Teach and learn 	Information <ul style="list-style-type: none"> Gather and organize Evaluate Communicate 	Technology <ul style="list-style-type: none"> Use Choose Maintain
Worker Role	Four arithmetic operations, estimation, geometry, algebra, exponential functions, spreadsheets, conversions. Concept of trade-offs. Awareness of tools such as linear programs and calculus for making trade-off decisions.	Model-building. Concept of first and second derivative and of integral, average, and standard deviation.	Mental arithmetic, fractions, percentages.	Create and read graphs, tables, and explanatory text.	Read graphs, tables, and explanatory text. Concept of trade-offs. Geometry.
Consumer Role	Four arithmetic operations, geometry, exponential functions, spreadsheets. Concept of trade-offs.	Concept of first and second derivative and of integral, average, and standard deviation.	Mental arithmetic, fractions, percentages.	Read graphs, tables, and explanatory text.	Read graphs, tables, and explanatory text. Geometry.
Citizen Role	Four arithmetic operations, geometry, concept of trade-offs.	Concept of first and second derivative and of integral, average, and standard deviation.	Mental arithmetic, fractions, percentages.	Read graphs, tables, and explanatory text.	Read graphs, tables, and explanatory text. Geometry.
Personal Role	Geometry, concept of trade-offs.	Concepts of calculus and statistics. History of mathematical discovery.		Read graphs, tables, and explanatory text.	Geometry.

Source: Packer, A. (2003). What mathematics should "everyone" know and be able to do? In B. Madison & L. Steen (Eds.), *Quantitative literacy: Why numeracy matters for schools and colleges* (pp. 33–42). Princeton, NJ: National Council on Education and the Disciplines.

- Ratio and proportion were used extensively in two settings and appeared in both staffing decisions and work tasks (i.e., dosage, conveyor belt speed). Such numbers were recorded regularly and considered in performance and quality monitoring.
- Formulae were used by a range of workers, but also were posted conveniently, so employees did not have to rely on memory. There was little evidence that people understood or cared about the mathematics reasoning behind the formulae.
- Most workers were involved in collecting and entering data. While they may not have been the employees who analyzed the data, they were expected to understand the implications drawn from the data when data were presented at meetings.

3. Forman & Steen (1999) described “Functional Mathematics” as the mathematics required for the technical and problem-solving needs of contemporary workplaces, for the modern demands of active citizenship, and for further education. The guiding principle of Functional Mathematics is the use of authentic applications. Content areas forming the basis of Functional Mathematics (envisioned as a reorganization of high school mathematics content) include topics found in traditional high school programs, such as “percentages and ratios; linear and quadratic equations; areas, angles, and volumes; and exponential growth and trigonometric relations,” but also less common topics, such as “index numbers, tolerances, three-dimensional geometry, indirect measurement, financial mathematics.” Additional topics such as “spreadsheets, data analysis and statistical quality control” should also be added (pp. v–vi).

The many case studies of workplace mathematical activity cited in this report stress that the particular context of the workplace has an important impact on the nature of effective mathematical performance. For example, Masingila (1994) studied carpet layers engaged in conceptually deep mathematical thinking as they solved problems encountered during installations. These constraint-filled situations differed substantially from straightforward textbook area problems. The carpet layers were able to accommodate situational constraints, such as a column in a room, while minimizing the number of seams and accounting for the nap in the carpet. They envisioned multiple ways to plan the carpeting and could choose among them for the best solution. Ninth-grade students who had studied area and had the procedural knowledge to solve the problems had difficulty recognizing that concepts of area were relevant here. They had difficulty addressing the constraints and could generate only one way to install the carpet (Masingila, Davidenko, & Prus-Wisniowska, 1996).

4. Wedege (2000, p. 131) has argued that math knowledge does not qualify someone for work unless it is integrated with knowledge, skills, and properties relevant to the practices and organization of the workplace. Within these contexts, proficient performance requires:
 - An understanding of relevant mathematical concepts.
 - The ability to make decisions about what mathematical concepts should be applied in various well- or ill-defined situations, recognizing and taking account of constraints.
 - The ability to use estimation and mental math skills, making decisions about the level of accuracy appropriate for the situation.
 - The ability to use technology as a mathematical tool (i.e., calculator, spreadsheet), determining when and how it should be used and interpreting results.

APPENDIX D: MATHEMATICAL CONTENT TOPICS FOR THE WORKPLACE

5. In the twenty-first century workplace, more complex or hybrid skills, such as the combination of technical and analytic knowledge with the ability to communicate analytical information, are increasingly in demand. Hoyles, Wolf, Molyneux-Hodgson, and Kent (2002) suggest that “techno-mathematical literacy” needed for work may go beyond computational skills. Techno-mathematical literacy skills include:
- Integrated mathematics and IT skills;
 - Ability to create a formula (using a spreadsheet if necessary);
 - Calculating and estimating (quickly and mentally);
 - Proportional reasoning;
 - Calculating and understanding percentages correctly;
 - Multi-step problem solving;
 - Sense of complex modeling, including understanding thresholds and constraints;
 - Use of extrapolation;
 - Recognition of anomalous effects and erroneous answers when monitoring systems;
 - Ability to perform paper-and-pencil calculations and mental calculations, as well as calculating correctly using a calculator;
 - Communicating mathematics to other users and interpreting the mathematics of other users; and
 - Ability to cope with the unexpected (Hoyles et al., 2002, p. 5).

Appendix E: Developmental Math Courses in Community Colleges

Although wide variation among community colleges makes it difficult to generalize, the following description is illustrative of a web review of college catalogues. Developmental mathematics offerings generally include a three-semester sequence of courses (arithmetic, introductory algebra, and intermediate algebra) aimed at preparing students for the basic required college mathematics course. The most basic developmental course prepares students for introductory algebra, covering a range of arithmetic-related topics and problem-solving skills. These include basic percent and proportion problems, basic systems of weights and measures, geometry of plane figures, arithmetic of signed numbers, and linear equations with one variable. The two additional developmental courses are introductory algebra, approximating a high school algebra I course, and intermediate algebra, approximating a high school algebra II course. These three courses do not replace any degree requirement or elective.

In their literature review, Golfin et al. (2005) examined four standards documents that provide a consistent, comprehensive view of the required foundational content knowledge (specific math facts or topics), necessary skills (critical thinking, generating ideas, and determining the tool needed to do a job), and abilities (attributes affecting the ability to perform a task, such as manual dexterity and inductive and deductive reasoning) needed to pursue college-level mathematics and career paths based on postsecondary certification programs.

APPENDIX E: DEVELOPMENTAL MATH COURSES IN COMMUNITY COLLEGES

Of these four documents, the standards developed by the American Mathematical Association of Two-Year Colleges (AMATYC, 1995) are most directly applicable to developmental mathematics course content. AMATYC extended its description of developmental mathematics courses in *Beyond Crossroads: Implementing Mathematics Standards in the First Two Years of College* (2006), suggesting that in developmental courses “some algebraic topics, such as factoring, radicals, and operations with rational expressions, should receive less attention, while modeling, communication, and quantitative literacy and reasoning should receive more attention” (p. 42). It is unclear how widely these recommendations have been implemented.

After completing the developmental sequence (or by achieving a sufficiently high CPT score), students reach college-level credit mathematics courses. There are different mathematics requirements for STEM (science, technology, engineering, and mathematics) and non-STEM degrees. Courses required for scientific fields include college algebra, calculus, and/or statistics. For humanities majors, survey courses such as contemporary mathematics may satisfy a mathematics requirement.

A quantitative literacy course also would likely satisfy a mathematics requirement for non-science majors. Lynn Arthur Steen, a leader in the Quantitative Literacy movement, has suggested on behalf of the Quantitative Literacy Design Team (2001) that the following skills, embedded in situational contexts, embody Quantitative Literacy:

- **Arithmetic:** Having facility with simple mental arithmetic; estimating arithmetic calculations; reasoning with proportions; counting by indirection (combinatorics).
- **Data:** Using information conveyed as data, graphs, and charts; drawing inferences from data; recognizing disaggregation as a factor in interpreting data.
- **Computers:** Using spreadsheets; recording data; performing calculations; creating graphic displays; extrapolating; fitting lines or curves to data.
- **Modeling:** Formulating problems; seeking patterns and drawing conclusions; recognizing interactions in complex systems; understanding linear, exponential, multivariate, and simulation models; understanding the impact of different rates of growth.
- **Statistics:** Understanding the importance of variability; recognizing the differences between correlation and causation, between randomized experiments and observational studies, between finding no effect and finding no statistically significant effect (especially with small samples), and between statistical significance and practical importance (especially with large samples).

APPENDIX E: DEVELOPMENTAL MATH COURSES IN COMMUNITY COLLEGES

- **Chance:** Recognizing that seemingly improbable coincidences are not uncommon; evaluating risks from available evidence; understanding the value of random samples.
- **Reasoning:** Using logical thinking; recognizing levels of rigor in methods of inference; checking hypotheses; exercising caution in making generalizations (pp. 16–17).

Because of the low success rates for students who take developmental mathematics courses in community colleges, initiatives are underway to rethink and reform the developmental mathematics sequence. In 2010, for example, the Carnegie Foundation for the Advancement of Teaching is working with several community colleges to develop courses that accelerate readiness for STEM courses. One Carnegie-sponsored course, Statway,¹ is designed to enable students currently referred to elementary algebra to complete a credit-bearing, transferable statistics course in one year, so that they are better equipped for the statistics required by STEM-related majors. Another one-semester course, Mathway, is intended to develop the foundations of mathematical literacy and decision-making. Upon completing the Mathway course, students will be prepared to take “various credit-bearing, transferable mathematics courses, including quantitative reasoning or mathematics for liberal arts, statistics, or college algebra.”

¹ <http://carnegiefoundation.org/statway/reconsider-and-redesign-developmental-mathematics>.

Appendix F:

Content of College Placement Tests

Math Content for the ACCUPLACER® Tests

The ACCUPLACER® sample question publication, geared toward students, includes detailed information about the ACCUPLACER® battery of tests. This information includes descriptions detailing the content covered in the ACCUPLACER Arithmetic, Elementary Algebra and College-Level Mathematics assessments.

The material below is an excerpt taken from a College Board publication titled “ACCUPLACER® Sample Questions for Students (Revised December 2007).”

Located on the College Board website, this publication can be accessed by visiting the following URL: <http://professionals.collegeboard.com/profdownload/accuplacer-sample-questions-for-students.pdf>.

Arithmetic

This test measures your ability to perform basic arithmetic operations and to solve problems that involve fundamental arithmetic concepts. There are 17 questions on the arithmetic test, divided into 3 types.

- Operations with whole numbers and fractions: Topics included in this category are addition, subtraction, multiplication, division, recognizing equivalent fractions and mixed numbers, and estimating.
- Operations with decimals and percents: Topics include addition, subtraction, multiplication, and division with decimals. Percent problems, recognition of decimals, fraction and percent equivalencies, and problems involving estimation are also given.

APPENDIX F: CONTENT OF COLLEGE PLACEMENT TESTS

- Applications and problem solving: Topics include rate, percent, and measurement problems; simple geometry problems; and distribution of a quantity into its fractional parts.

Elementary Algebra

This test contains a total of 12 questions of 3 types.

- Operations with integers and rational numbers, including computation with integers and negative rationals, the use of absolute values, and ordering.
- Operations with algebraic expressions using evaluation of simple formulas and expressions, and adding and subtracting monomials and polynomials. Questions involve multiplying and dividing monomials and polynomials; evaluation of positive rational roots and exponents; simplifying algebraic fractions; and factoring.
- Translating written phrases into algebraic expressions and solving equations, inequalities, word problems, linear equations and inequalities, quadratic equations (by factoring), and verbal problems presented in an algebraic context.

College-Level Mathematics

The college-level mathematics test measures your ability to solve problems that involve college-level mathematics concepts. This test has a total of 20 questions in 6 content areas:

- Algebraic Operations
- Solutions of Equations and Inequalities
- Coordinate Geometry
- Applications and Other Algebra Topics
- Functions
- Trigonometry

Algebraic Operations includes the simplification of rational algebraic expressions, factoring and expanding polynomials, and manipulating roots and exponents. *Solutions of Equations and Inequalities* includes the solution of linear and quadratic equations and inequalities, systems of equations, and other algebraic equations. *Coordinate Geometry* covers plane geometry, the coordinate plane, straight lines, conics, sets of points in the plane, and graphs of algebraic functions. *Applications and Other Algebra Topics* includes complex numbers, series and sequences, determinants, permutations and combinations, factorials, and word problems. *Functions* includes questions involving polynomial, algebraic, exponential, and logarithmic functions. *Trigonometry* addresses trigonometric functions.

Math Content for the COMPASS® Tests

Numerical Skills/Pre-Algebra Placement Test

Content for questions in the Numerical Skills/Pre-Algebra Placement Test ranges from basic arithmetic concepts and skills to the knowledge and skills considered pre-requisites for a first algebra course. The Numerical Skills/Pre-Algebra Placement Test includes items from more than a dozen content areas; most questions, however, come from the following categories:

1. Operations with Integers
2. Operations with Fractions
3. Operations with Decimals
4. Positive Integer Exponents, Square Roots, and Scientific Notation
5. Ratios and Proportions
6. Percentages
7. Averages (Means, Medians, and Modes)

Algebra Placement Test

The Algebra Placement Test is composed of items from three curricular areas: elementary algebra, coordinate geometry, and intermediate algebra. Each area is further subdivided into several specific content areas. Overall, the Algebra Placement Test includes items from more than 20 content areas; most questions, however, fall within the following 8 content areas:

1. Substituting Values into Algebraic Expressions
2. Setting Up Equations for Given Situations
3. Basic Operations with Polynomials
4. Factoring Polynomials
5. Linear Equations in One Variable
6. Exponents and Radicals
7. Rational Expressions
8. Linear Equations in Two Variables

These descriptions were drawn from <http://www.act.org/compass/sample/pdf/numerical.pdf>.